WORKSHOP SCIENCE

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A PRELIMINARY COURSE FOR TECHNOLOGICAL STUDENTS

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PREFACE

It is now generally recognised that evening students, proposing to follow a course in science or technology at a technical school, require some preliminary training in elementary general science. It is important that, in providing this training, the final aim of the student should be kept in view; and, from the first, the various applications of scientific principles to workshop practice should be pointed out. From this point of view the ordinary science text-book is unsuitable, and an attempt is here made to supply the deficiency.

It is usual to provide both lecture demonstrations and practical work in this elementary science, and the author has felt that, in consequence of the inability of many evening students to take good notes, a book dealing systematically with the subject-matter of the lectures would be useful; whereas in the practical work the student should make his own notes as he goes along. Some suggestions for practical exercises are included, but as the type of apparatus and equipment varies in different laboratories, details have been left to the teacher.

The Union of Lancashire and Cheshire Institutes includes the subject in the examination for the Preliminary Technical Certificate, and the subject-matter of their syllabus is fully covered in the book.

Some of the more difficult sections might advantageously be omitted on a first reading, except in the case of more advanced students.

Certain of the diagrams, by the courtesy of the author and publishers, have been borrowed from Dr. Wormell's books on Mechanics and Heat.

The author also desires to acknowledge with thanks the useful suggestions and assistance in revising proofs which he has received from his colleagues, Mr. J. H. Moore, B.Sc., Mr. M. J. H. Cooke, B.Sc., and Mr. R. Chorley, A.R.C.Sc.

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CHAPTER I

MATTER AND FORCE

Matter.—Every one knows that various things are made of different kinds of stuff or material. The general name of **matter** is given to all the different kinds of stuff.

The following are examples of some different *kinds of matter* or *substances* :—

(*a*) Wood, stone, iron, bone, glass, ice.
(*b*) Water, milk, oil, spirit.
(*c*) Air, coal-gas, steam.

It will be observed that the substances in the first set (*a*) are what are commonly called **solids**; those in the second set (*b*) are **liquids**; and those in the third set (*c*) are **gases.**

The student should note that these three names do not necessarily represent different kinds of matter; thus, ice differs no more from water than a piece of lead differs from the same lead when melted; in fact, ice, water, and steam are not different substances, but simply different states or conditions of the same kind of material.

Force.—We constantly see changes taking place in the things around us; these changes are caused by certain influences called **forces.**

Thus, if a stone be unsupported, it falls to the ground, *i.e.* it undergoes a change of position, because it is attracted to the earth by a force called **gravity.** The particles of the stone are held together by another force called **cohesion.** A piece of lead may be bent or changed in shape by the application of muscular force. The same piece of lead may be melted or changed to a liquid by the force of **heat.** A quantity of gunpowder, on the application of a light, is instantly changed into smoke and invisible gases by the action of **chemical force.** Again, a piece of iron exposed to damp air is gradually changed into another substance called rust, also by the action of chemical force.

A force may thus be defined as an influence which tends to produce changes in matter, or it may, as in the case of cohesion, tend to resist changes.

A further discussion of force will be found in a later chapter.

Properties of Matter.—Different substances produce different effects on account of the different **properties** which they possess.

Thus, sugar produces a certain effect on our sense of taste because it has the property of sweetness. A piece of red cloth produces a special effect on our sense of sight on account of its property of redness. A lump of lead produces certain effects because it has the property of weight. Water produces different effects from ice, because it has the properties of a liquid, and so on.

There are certain general properties which are possessed by all kinds and all forms of matter. Thus, **all matter has weight,** and **all matter occupies space.**

Every one knows that solids and liquids have weight, but it is not so clear in the case of gases. We may, however, easily show that air has weight by taking a fairly large glass globe tightly closed and provided with a tap. If we carefully balance this on a pair of good scales and then pump the air out and close the tap, we shall find that it has become lighter; if we open the tap and let the air rush in again it will regain its original weight. It can be shown, in fact, that a cubic foot of air weighs about $1\frac{1}{4}$ ounces, and thus the air in a room 12 feet square by 10 feet high would weigh $12 \times 12 \times 10 \times 1\frac{1}{4} = 1800$ ounces, or $112\frac{1}{2}$ lbs.; *i.e.* about one hundredweight.

Coal-gas is lighter than air, and that is why a balloon filled with coal-gas tends to rise. There are other gases heavier than air.

Again it is evident that solids and liquids occupy space or take up a certain amount of room. Thus, if we take a glass full of water and drop a stone into it, some of the water will overflow, because it is pushed out by the stone; the same space cannot be occupied by the water and the stone at one and the same time.

If we take a glass tumbler and push its mouth downwards under water, we shall find that the water does *not* enter the tumbler, because the space is already fully occupied by the air which it contained. If we gradually tilt the tumbler to one side we shall find that bubbles of air escape and rise through the outer water, and, as the air gets out, the water is able to get into the tumbler.

In addition to these *general* properties possessed by all kinds of matter, there are certain *special* properties peculiar to different states or conditions of matter.

Solids.—A solid has a definite shape and size, and it offers more or less resistance to any change of the shape or of the size.

Thus, if you take a rod of iron and squeeze it so as to try to make it thinner or shorter, or pull it, to try and lengthen it, or if you try to bend it, or to twist it, you will produce no perceptible effect, either on its size or on its shape. But we know that the iron can be bent or twisted, if we exert sufficient force; and it can even be *slightly* lengthened or shortened by the application of great force. This resistance to change of shape is called **rigidity.**

Some solids are less rigid than others; thus, a piece of lead is more easily bent than iron, and it is easily flattened by hammering. When wrought iron is made hot, it becomes softer and less rigid and can then be hammered and rolled into different shapes.

Different solids behave differently under the action of forces tending to change their shape; thus, if we take a thin rod of cast iron and try to bend it, it will break rather than bend, because it is *brittle:* whereas a similar rod of wire of wrought iron will bend without breaking, being more or less *pliable or flexible;* and again, a wire of tempered steel will bend when we subject it to force, but, as soon as we release it, it will spring back to its original form, because it is *elastic.*

Again, solids offer resistance to being divided into parts, because of the strong force of cohesion Some offer more resistance than others. Thus, a piece of wood is easily cut with a knife, a piece of brass can be cut, but less easily, while on a piece of glass, on account of its great *hardness,* the knife makes no impression at all. The glass is, however, readily broken by a blow on account of its great *brittleness.* The property of hardness is made use of in grindstones and emery-wheels, used for sharpening tools: the stone, being harder than the metal, rubs or scratches particles off the latter.

Some solids will withstand one kind of force better than another. Thus, a cast-iron pillar will support a great weight, but is readily shattered by a blow, or snapped by a bending force. A wrought-iron wire or a rod offers little resistance to bending, but will support a considerable weight suspended from it, on account of its great tenacity. Masonry and brickwork will readily support a dead weight, but should not be subjected to a great bending force; thus, stone is not suitable for making girders or joists. Some solids are liable to change their properties when in use; thus, a piece of iron wire, when bent backwards and forwards a number of times, becomes weaker and will finally break; and steel rails and carriage wheels are liable to become brittle on account of the continual vibration.

Although solids offer a more or less strong resistance to change of shape they may often be altered in shape by suitable means. Some solids, like wet clay and putty, are *plastic*, and can easily be moulded by the fingers or by suitable tools. Others, like lead, can be gradually beaten into shape, or, by very great pressure, can be forced into the required shape, as in making solid-drawn lead pipes. Others, again, like wrought iron, and copper, and gold, are *malleable*, as they can be beaten, or hammered, or rolled into thin sheets; and they are also *ductile*, so that they can be drawn through smaller and smaller holes, forming thinner and thinner wire. Lead is not readily drawn into wire because, although soft and yielding, it is not sufficiently tenacious to stand the pulling force. Wood, on account of its fibrous structure, behaves differently according as force is exerted along or across the grain. Thus it is easily split, or shaved, or broken, in the direction of the fibres, but it requires much greater force to break or cut across the fibres; hence, joists should, of course, always be cut so that the length is parallel to the fibres of the wood.

Liquids.—The chief property of liquids is their **fluidity,** or power of flowing, which depends on the fact that the different parts, instead of being firmly bound by the force of cohesion, as in the case of solids, are free to move about amongst each other. In consequence of this, a liquid has no particular shape of its own, but adapts itself to the shape of the containing vessel, and may be poured from one vessel to another, either wholly or in part; for a liquid offers no resistance to division into parts. Another consequence of this power of flowing is that the free upper surface of a liquid, when at rest, is flat and level; for, if any portion stood up above the rest, its weight would cause it to flow to the lower parts until it found its level. Different liquids possess this property of fluidity in different degrees; thus, water flows more readily than treacle or glycerine, but it flows less readily than spirit—hence spirit is preferred in a spirit-level. Liquids which flow sluggishly, like treacle and glycerine, are said to be viscous, or to possess the property of **viscosity.** This property is possessed in a high degree by certain oils, used for lubricating purposes. These thick or viscous oils are specially suitable for heavy, slow-moving machinery, such as engine and flywheel bearings, whereas the thinner oils are better adapted for light and quickly-moving parts, *e.g.* sewing-machines and the spindles of cotton machinery. If a thin oil were used on a heavy bearing, it would be squeezed out, and the bearing would soon run dry, and would then get hot.

The property, which liquids have, of adapting themselves to the shape of any vessel or receptacle is made use of in the casting of metals, the molten metal being run into suitable moulds, so that when it becomes solid it retains the shape of the mould.

Although a liquid readily changes its shape, it offers great resistance to change of size or volume. Thus, you cannot squeeze a quart of liquid into a vessel which is even slightly smaller than a quart. Liquids, in other words, are practically incompressible; thus, if you press very hard on a liquid in a closed space, the liquid will not yield, but will press equally hard against the sides of the vessel; hence, a liquid may be used to transmit great pressures, as in the case of hydraulic machinery. Liquid pressure is discussed more fully in a later chapter.

Gases.—A gas, such as air, resembles a liquid in having the property of fluidity. Thus, winds and draughts are examples of air which is flowing or moving along.

Again, the gas supplied from the gasworks flows along the pipes; and steam flows from the boiler to the cylinders of an engine. Gases have the further property of diffusing or spreading out automatically; thus, if some coal-gas escapes in one part of a room, it can soon be smelt all over the room. Gases differ from liquids in being, as a rule, invisible, and also in being generally much lighter than liquids. Thus, whereas a cubic foot of water weighs about 1000 ounces, a cubic foot of air only weighs about $1\frac{1}{4}$ ounces.

But the most important difference between liquids and gases is that gases are **compressible**, *i.e.* they can be squeezed into a smaller space, and they are also **elastic**, *i.e.* they will expand and spread over a much larger space if the pressure is reduced.

The compressibility and elasticity of air are readily illustrated by taking a *stout* glass tube about 8 or 9 inches long, and, say, $\frac{3}{8}$ or $\frac{1}{2}$ an inch diameter, with sharp edges, by means of which it is easy to cut two plugs from a slice of potato about $\frac{1}{2}$ an inch thick. These two plugs enclose a tube full of air, and if we place one end on the table and press down the upper plug with a thick pencil, it will be found that the air can be readily compressed into a smaller bulk. If we suddenly let go, the elasticity of the air causes the plug to spring back, and if it fits well, it often shoots the pencil some distance into the air. The same properties can be illustrated by a bicycle pump, the nozzle being closed to prevent the air escaping. When a tyre is pumped up, we force in much more air than would fill the tyre at the ordinary pressure.

It is the elasticity of steam which causes it to rush from the boiler to the cylinder of an engine, and also to continue expanding in the latter after the supply from the boiler has been cut off by the valve.

The elasticity and pressure of gases will be further discussed in a later chapter.

SUMMARY

Matter exists in three states: solid, liquid, and gaseous.

All the three forms have weight and occupy space.

Solids have a definite shape, *i.e.* they are more or less rigid.

Liquids are fluid, *i.e.* they have the power of flowing, but they are not compressible. Viscous liquids flow sluggishly.

Gases are fluid, and they are also compressible and elastic.

CHAPTER II

MENSURATION OR MEASUREMENT

The Metric System—Measurement of Area

IN order that we may be able to measure and compare different objects as regards length, area (extent of surface), volume (space occupied), and weight, it is necessary that we should have certain standards or units of measurement, and these standards must be of the same kind as the quantities we wish to measure; thus, the standard of length must be a length, the standard of area must be an area, and that of weight a weight, and so on. It is convenient to have several units of each kind for different purposes. Thus, the British standard of length is the yard; but we also have the mile (or 1760 yards) for large measurements, and the foot and inch (or $\frac{1}{36}$ yard) for small measurements. The British system of units is complicated, and leads to clumsy methods of reduction; hence students should know something of the continental system originated in France.

The Metric System.—The great feature of this system is that it is a *decimal* system, *i.e.* it is based upon the number 10. This number is much the easiest to work with, on account of its being also the basis of our system of counting and writing of numbers.

The fundamental unit, from which all the others are derived, is a unit of length, called the "**metre**," which is equal to 39·37 inches, and therefore about $3\frac{3}{8}$ inches longer than a British yard.

For larger measurements multiples by 10, 100, and 1000 are used, and for small measurements there are fractions of the metre, viz. $\frac{1}{10}$ or ·1, $\frac{1}{100}$ or ·01, and $\frac{1}{1000}$ or ·001.

These multiples and fractions have special names, as shown in the following table; but only those shown in heavy type are in common use.

10 metres = 1 dekametre.
100 metres = 1 hektometre.
1000 metres = 1 **kilometre.**
$\frac{1}{10}$ or ·1 metre = 1 **decimetre.**
$\frac{1}{100}$ or ·01 metre = 1 **centimetre.**
$\frac{1}{1000}$ or ·001 metre = 1 **millimetre.**

It is usual to employ only one denomination for any one measurement. Thus, instead of writing—

2 metres, 3 decimetres, 5 centimetres, 4 millimetres,
we should write 2.354 metres,
or 235.4 centimetres.

This example also serves to illustrate the fact that reduction from one denomination to another is effected by simply moving the decimal point to right or left, as it is simply a case of multiplying or dividing by 10 or 100 or 1000, &c.

The units of area are, of course, easily derived from those of length; thus we have the square metre and square centimetre, &c. The student should note that whereas 10 centimetres are equal to 1 decimetre, it takes 100 square centimetres to form a square decimetre, just as it takes 144 square inches to make 1 square foot, and a similar rule applies in other cases.

The units of volume or capacity are also easily derived from those of length; thus we have the cubic metre, the cubic centimetre, and so on.

In measuring liquids a special name is given to the cubic decimetre as a measure of capacity: it is called the "**litre**."

It is most important to note that **1 litre = 1000 cubic centimetres.**

For 1 decimetre = 10 centimetres.
1 sq. decimetre = 100 sq. centimetres.
1 cubic decimetre = 1000 cubic centimetres.

The cubic centimetre is much used for small measurements, and the name is often abbreviated to c.c.

The standard of weight is derived from the standard of volume in a very simple way, thus:—

1 **gram** is the **weight** of 1 **c.c.** of **water.**[1]

This is very convenient, because it means that the volume of any quantity of water, expressed in c.c., and its weight, expressed in grams, will be represented by the same number; thus, the weight is at once known from the volume, and *vice versâ*. This, of course, applies only to water; but other substances are easily dealt with if we know how many times they are heavier or lighter than water (see next chapter).

Other units of weight are distinguished by the same prefixes as in the units of length:—

10 grams = 1 dekagram
100 grams = 1 hektogram.
1000 grams = 1 **kilogram.**

$\frac{1}{10}$ or ·1 gram = 1 decigram.
$\frac{1}{100}$ or ·01 gram = 1 centigram.
$\frac{1}{1000}$ or ·001 gram = 1 milligram.

Weights are, however, generally expressed in grams or kilograms and decimals of a gram or kilogram.

British and Metric Equivalents.—It is convenient to remember the following rough equivalents:—

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[1] The water must be measured at a particular temperature, viz. 4° C., its temperature of maximum density (see chapter xvi.).

British.		*Metric.*
39·37 inches	=	1 metre.
1 inch	=	2·54 centimetres.
4 inches	=	10 centimetres (about).
5 miles	=	8 kilometres (about).
1 ounce	=	28·35 grams.
2·2 lbs.	=	1 kilogram.
1 gallon		4·54 litres.
7 quarts	=	8 litres (about)

Area.—By area we mean the extent of surface. It is measured by square measure (*e.g.* in square yards, square miles, or square inches, or in square metres, square kilometres, or square centimetres) although the surface to be measured may have any shape, *i.e.* it need not be square.

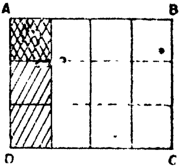


FIG. 1

A four-sided figure with square corners (like the figure) is called a rectangle. Its area is readily found by imagining it to be divided into square inches or square centimetres as the case may be. Suppose the length AB = 4 inches and breadth AD = 3 inches. Then it may be divided by vertical lines into 4 strips (like the shaded portion) each 1 inch wide. Each strip may be divided by cross lines into 3 squares (like the double-shaded portion) measuring 1 inch each way and therefore equal to a square inch. Hence the total number of square inches will be 4 × 3 = 12. Similarly the area of any rectangle may be found by multiplying the numbers representing the length and breadth, but we must be careful that both measurements are expressed in the same denomination. Thus the area of a rectangle 1 foot long by 1 inch wide is not 1 square inch or 1 square foot, but 12 × 1 = 12 square inches or $1 \times \frac{1}{12} = \frac{1}{12}$ square foot.

A three-sided figure like ABC is called a triangle. AB may be called its base and the dotted line CD, perpendicular to the base, its height. We may construct a rectangle ABEF having the same base and the same height as the triangle. It is easy to see that this rectangle has just twice the area of the triangle, for the portion of the triangle ACD just equals ACF and CDB just equals CBE; thus the given triangle forms one half of the rectangle and the shaded parts form the other half. Now the area of the rectangle = length × breadth. But the length is the length of the base of the triangle and the breadth equals the height of the triangle. Hence the area of the rectangle equals base of triangle × height of triangle. Therefore **area of triangle = $\frac{1}{2}$ base × height**.

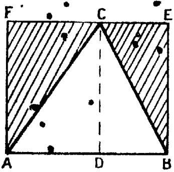


FIG. 2

Any surface whatever, whose boundaries are straight lines, may be divided up into rectangles and triangles,

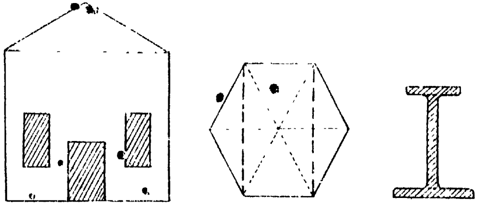


FIG. 3.

and its area will be equal to the sum of these; consequently the above rules will serve to find the area of any figure bounded by straight lines. The diagrams suggest some applications of the rules, the dotted lines showing the method of dividing into rectangles and triangles.

The Circle.—A line drawn from the centre of a circle to the circumference is called a **radius**, *e.g.* AO, BO, CO, and DO.

All radii of the same circle are **equal**.

A double radius like AOB going straight through the centre is called a **diameter**.

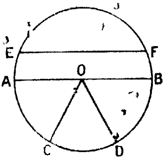


FIG. 4.

Any line like EF going across the circle, but not through the centre, is called a **chord**.

All diameters are, of course, equal, and measure double the radius. A diameter is greater than any chord not passing through the centre, and hence the diameter may be considered as the greatest distance across a circle. On this fact depends the use of callipers for finding the diameter of a cylinder or of a disc (e.g. a coin). If callipers are not available, a couple of set squares or square blocks laid against the scale with the circle between them will do quite well.

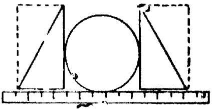


FIG. 5.

The relation between the diameter of a circle and the circumference (or measurement all round the boundary) is very important.

The student should make a series of experiments both with large circles (*e.g.* wheels, large cylindrical jars, &c.) and with small circles (*e.g.* coins and small wooden or metal cylinders), tabulating the results thus :—

Object.	Diameter.	Circumference.	Circumference / Diameter
Halfpenny . .	1 inch	3·14 inches	3·14 inches
Cycle wheel . .	28 inches	88 inches	3·14 inches

When a wheel rolls round once, it travels forward by a distance equal to its circumference. Thus the circumference of a coin can be found with fair accuracy by making a small ink spot on the rim and rolling it along a straight line so as to make two ink marks on the paper; the distance between the marks will be the circumference of the coin.

It will be found that the relation between circumference and diameter is the same in all circles whether large or small, viz.—

Circumference = $3\frac{1}{7}$ or 3·1416 times the diameter.

This number, which is not exactly $3\frac{1}{7}$, nor exactly 3·1416, is often denoted by the Greek letter π (*pi*).

Thus if C be the circumference and r the *radius* of a circle, $C = 2\pi r$.

Area of Circle.—By drawing radii we may divide a circle into any number of triangles with curved bases.

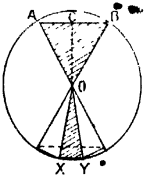


FIG. 6.

If we make a wide triangle like AOB we note:

(1) That there is a considerable part of the circle left between the straight and curved lines AB.

(2) That the height OC of the triangle is less than the radius of the circle.

(3) That the base AB is less than the curved part of the circle AB.

If we divide a similar triangular space into three smaller spaces we find that the narrow triangle XOY approaches much more nearly to the corresponding part of the circle in all the respects 1, 2, and 3 mentioned above.

Now we can, in imagination, divide the circle into as many triangles as we like, and it is reasonable to conclude that if we make the number large enough, the whole series of very narrow triangles will *practically* include the whole area of the circle.

Further, the height of each triangle will equal the radius of the circle,

and the sum of the bases of all the triangles will equal the circumference.

Thus, we have :—

Area of each triangle $= \frac{1}{2}$ base $\times$ height $= \frac{1}{2}$ base $\times$ radius.

Area of all the triangles $= \frac{1}{2}$ (sum of all bases) $\times$ radius.

$= \frac{1}{2}$ circumference $\times$ radius

$= \frac{1}{2}$ of $2\pi r \times r$.

$= \pi r \times r$ or $3\frac{1}{7}r^2$.

It is interesting to note that r^2 will represent the area of a square whose side equals the radius of the circle, and that $3\frac{1}{7}$ times the area of this square represents the area of the circle. This is shaded in the diagram.

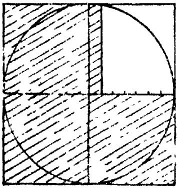


FIG. 7

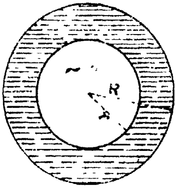


FIG. 8

The area of a ring-shaped surface is easily found as the *difference* in area between the two circles forming its inner and outer boundaries. Thus, if the radii be R and r we have :—

Area of large circle $= \pi R^2$.

Area of small circle $= \pi r^2$.

Difference $=$ area of ring $= \pi R^2 - \pi r^2 = \pi(R^2 - r^2)$;

i.e. area of ring equals $3\frac{1}{7}$ times the difference of the squares of the inner and outer radii.

SUMMARY.

The metric system is a decimal system, *i.e.* based on number 10.

A metre = 39·37 inches = 1 yard plus 3⅓ inches.

A kilometre = 1000 metres (about ⅝ mile).

A litre = 1 cubic decimetre = 1000 cubic centimetres (1000 c.c.).

A gram = the weight of 1 c.c. of water.

A kilogram = 1000 grams (about 2⅕ lbs.).

Area of rectangle = length × breadth.

Area of triangle = base × ½ height.

Circumference of circle = 3⅐ times diameter = $2\pi r$.

Area of circle = 3⅐ times square on radius = πr^2.

EXERCISES.

1. How many millimetres are there in an inch?

2. By how much does a foot exceed 30 centimetres?

3. What weight of water will fill a tank whose capacity is 1000 litres?

4. A train travels 8 kilometres in 6 minutes. Find its speed in miles per hour.

5. A gable-end of a house measures 24 feet across and is 20 feet high to the eaves and 28 feet to the ridge. Find its area, allowing for two windows 3 feet 6 inches by 5 feet and a door 3 feet by 6 feet 6 inches.

6. A bicycle wheel has a diameter of 28 inches, and revolves twice a second. Find the speed of the machine in miles per hour.

7. Find the area of a circular path 7 feet wide surrounding a grass plot 21 yards in diameter.

PRACTICAL EXERCISES.

1. Measure several straight lines in centimetres and inches, using a rule divided into $\frac{1}{10}$ inch and estimating $\frac{1}{100}$ inch by eye. Find from your results the number of centimetres in an inch.

Find the relation between the circumference and diameter of large and small circles by various methods, such as the following:—

(*a*) Draw a circle of 3½ inches radius, and with a pair of dividers step off distances of ½ inch round the circumference. How many times should it go if the value of $\pi = 3\frac{1}{7}$? How many times does it actually go, and what fraction (if any) of ½ inch is left?

(*b*) Find, with callipers or set squares, the diameter of a wooden cylinder; then find its circumference by passing a narrow strip of paper tightly round it, and pushing a pin through to overlapping part, then measuring the distance between the two pin-pricks.

(*c*) Find the diameter of a cycle-wheel with tightly pumped tyres. Make a chalk mark on the tyre, and measure the distance along the floor between two traces left by the chalk when the wheel is rolled in a straight line.

(*d*) Apply the same method by rolling a coin with an ink spot on it along a piece of paper. (Take care to avoid any sliding motion.)

3. Find, by callipers, the diameter of various samples of glass rod, tubing, metal cylinders, spheres, &c.

4. Find the average diameter of small shot by arranging, say, 20 pellets, touching each other in a row and finding the total length.

5. Construct several rectangles whose sides are a whole number of centimetres long and divide into square centimetres. Repeat, using sides of, say, 3½ and 2½ centimetres, and by counting the whole, half, and quarter squares see whether the rule, length × breadth = area, holds good with fractional numbers.

6. Cut out to definite dimensions various figures—triangles, hexagons, circles, &c., in card or sheet metal of uniform thickness; weigh the figures, and compare with the weight of a square (say, of 5 centimetres or 2 inches side) of the same material, and calculate the areas of the

figures by proportion. Compare your results with the areas calculated from the dimensions.

7. Draw various regular and irregular figures on squared paper, and find the area by counting the squares, trying to balance the parts left out with equal parts added on, where the squares overlap the boundary.

CHAPTER III

MEASUREMENT OF VOLUME

By the volume of a body (whether solid, liquid, or gaseous) we mean the amount of space which it completely fills. If the body be a solid, it may, on account of its shape, take up room, without filling the space completely. Thus a chair or a table takes up a good deal of space in a room, but the space between the legs is not filled by the solid material of the chair. If we imagine the chair to be made of metal, which can be melted down and run into a compact mass, we shall then be able to form some idea of the true volume or space actually occupied by the material of the chair; for it must be clearly understood that the volume of a given quantity of the material will be the same, whatever be its shape, so that a mass of metal, when melted down, has just the same volume as it had before.

In measuring volume we must, of course, have a unit which is itself a volume. It is usual to derive this from the unit of length; thus, a cubic inch is the volume of a cube whose length, breadth, and thickness are each 1 inch, and similarly we may have a cubic metre and a cubic centimetre. It must be understood that although the unit is a cubical measure the volume to be measured need not be cubical in shape. Thus, if we have a cubic centimetre box and fill this with water, and then transfer the water to a round tube, we shall still have a cubic centimetre of water, although its shape is no longer that of a cube. Again, we have a solid ball, or a ring, or a body of any shape whose volume is a cubic inch; if the material of this ball or ring, &c., be melted down, it would just fill a cubical box measuring 1 inch each way.

Volumes of Regular Solids.—The volume of a rectangular solid like the one shown is easily found from the length, breadth, and thickness. Thus, suppose it to be 5 inches long, 4 inches wide, and 3 inches thick. It may be cut into 3 slices, each 1 inch thick. The top face ABCD may be divided into $5 \times 4 = 20$ squares (1 inch each way), as explained under areas, and if we cut down the top slice along the lines it will be divided into 20 cubes measuring 1 inch each way. Each of the three slices can be treated similarly, and hence we shall have $20 \times 3 = 60$ cubes, and as each cube measures 1 inch each way the total volume of the solid must be 60 cubic inches.

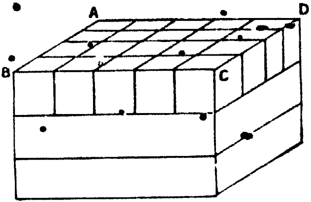


FIG. 9.

Thus the rule for finding the volume of a solid of this shape is to multiply the numbers which represent length, breadth, and thickness; but care must be taken that all three dimensions are expressed in the same units. Thus, if we have a box 3 feet long by 1 foot broad by 4 *inches* deep, the volume will be $3 \times 1 \times \frac{1}{3} = 1$ cubic foot (since 4 inches $= \frac{1}{3}$ foot) or $36 \times 12 \times 4 = 1728$ cubic inches.

The student may profitably test the rule for himself by building up solids with 1-inch cubes made of wood, or by drawing chalk lines on boxes so as to show how the space occupied by the body could be divided into cubes.

The method of dividing a solid into imaginary slices can be applied in a great many cases for the purpose of determining volumes.

For instance, all the forms illustrated in the diagram resemble each other in this respect that, if they be cut into slices, the size and shape of the slices will be the same all the way along, the shape of each slice being indicated by the shaded face in the diagram.

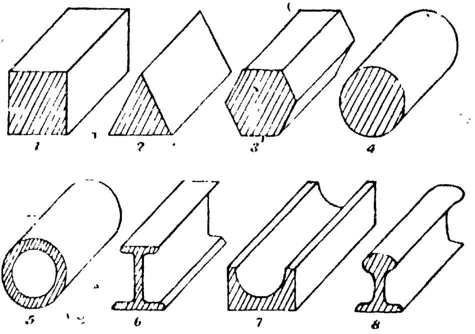


FIG. 10.

The forms 1, 2, and 3 are known respectively as square prism, triangular prism, and hexagonal prism, and No. 4 is a cylinder, which may be considered as a circular prism. The same method of working may be applied to all these regular forms, as well as to the remaining forms, 5, 6, 7, and 8, and any others of the same character.

If we are measuring in inches or centimetres, let us imagine each solid cut up into slices, 1 inch, or 1 centimetre in thickness; then evidently the number of slices will be the same as the number of units in the length of the solid.

Now, imagine the shaded face of each solid divided up into squares (square inches or square centimetres, as the case may be); then if we cut through the dividing lines, the end slice will be cut up into a number of cubes (cubic inches or cubic centimetres), just as many, in fact, as there are squares in the area.

Hence number of cubes in each slice = number of units of area in the end.

Therefore total number of cubes in all the slices
= number of slices × number of cubes in each slice
= number of units of length × number of units of area in the end.

The rule may be more briefly expressed thus:—

Volume of prism = length × area of base.

The cylinder is a most important solid, and it may be useful to state the special rule applied to it.

The area of the circular base $= \pi r^2$ or $3\frac{1}{7}r^2$.

Hence **volume of cylinder** $= \pi r^2 \times l$ or $3\frac{1}{7}r^2 \times l$ where l stands for length.

Example:—

To find the volume of 1 mile of telegraph wire $\frac{1}{4}$ inch thick.

Diameter of wire $= \frac{1}{4}$ inch, hence radius $= \frac{1}{8}$ inch.

Area of circular end $= 3\frac{1}{7} \times (\frac{1}{8})^2 = \frac{22}{7} \times \frac{1}{64}$.

Length of cylinder = 1 mile = 1760 yards = 1760 × 36 inches.

Therefore volume of wire $= 1760 \times 36 \times \frac{22}{7} \times \frac{1}{64} = 3111{\cdot}42$ cubic inches $= \frac{3111{\cdot}42}{1728} = 1{\cdot}8$ cubic feet.

Relation of Cylinder to Sphere and Cone.—If we have a hollow cylinder 1 inch in diameter and 1 inch high, it will just hold a ball or sphere 1 inch in diameter, as shown in the sketch, but of course the ball will only partly fill it. If we measure the quantity of water required to fill the cylinder when the ball is out, and again when the ball is in it, we shall find that in the latter case it just requires one-third as much water, hence the sphere occupies *two* thirds of the volume of the corresponding cylinder.

Now the volume of a cylinder $= \pi r^2 \times l$.

But if the cylinder is made to just hold a sphere of radius r the length of the cylinder must equal the diameter $= 2r$.

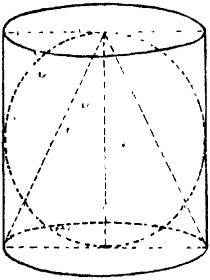


FIG 11

Hence volume of the cylinder $= \pi r^2 \times 2r = 2\pi r^3$. And volume of sphere $= \frac{2}{3}$ of cylinder $= \frac{4}{3}\pi r^3$.

If, instead of a sphere, we take a cone whose base and height just correspond to those of the given cylinder, we may show by displacement with water that the volume of the cone $= \frac{1}{3}$ volume of cylinder.

Hence volume of cone of height h and diameter $2r$ will be $\frac{1}{3}\pi r^2 h$.

The cone and sphere are shown dotted inside the cylinder in the diagram.

The points to remember are:—

Volume of sphere = $\frac{2}{3}$ volume of corresponding cylinder.

Volume of cone = $\frac{1}{3}$ volume of corresponding cylinder.

VOLUME OF LIQUIDS—AND OF SOLIDS BY DISPLACEMENT.

As liquids adapt themselves to the shape of the containing vessel, it is easy to construct vessels suitable for their measurement. The ordinary milkman's and spirit-dealer's measures are common examples, in which the measure is simply filled to the brim with the liquid.

The standard of liquid measure in this country is the gallon, which is simply the volume occupied by 10 lbs. of water. Since a cubic foot of water weighs about 62½ lbs., it follows that it takes about $6\frac{1}{4}$ gallons to make a cubic foot. A quart is simply a quarter of a gallon, and a pint is half a quart or one-eighth of a gallon, and hence a pint of water weighs $\frac{1}{8}$ of 10 lbs., *i.e.* $1\frac{1}{4}$ lb. or 20 ounces.

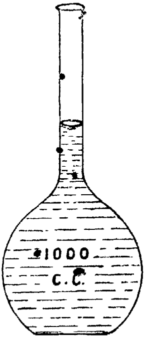
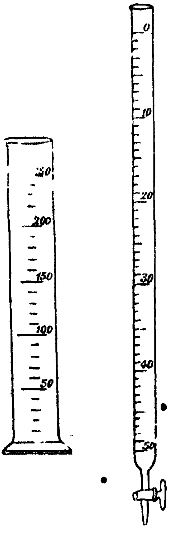


FIG. 12.



(*a*) (*b*)

FIG. 13.

In the metric system we have seen that the standard of volume is the litre, which is 1 cubic decimetre or 1000 cubic centimetres (1000 c.c.). In accurate measurements the vessel should not be arranged to be filled to the brim, because it is not easy to see exactly when it is just full: a considerable quantity of liquid can be added after it is full, without overflowing. It is much better to use a glass vessel with a narrow neck bearing a scratch on the neck to show the exact level to which it should be filled. The diagram represents a standard litre flask. The neck is made *narrow* because a drop or two more or less liquid will then make an appreciable difference in level, whereas in a wide vessel a few drops extra would not make a perceptible difference. Such a flask will, of course, only measure one quantity, and separate measures must be used for $\frac{1}{2}$ litre, $\frac{1}{4}$ litre, and so on. In many cases it is more convenient to use a tall, narrow measuring vessel with a series of marks to represent different volumes, as in the measuring cylinder (*a*) and the burette (*b*) in the diagram. It will be seen that in the cylinder the scale begins at the bottom and reads upwards, so that we can at once read off the volume of liquid in the cylinder. In the burette, on the other hand, the scale begins at the top and reads downwards, and the divisions *stop before the bottom*; thus we cannot tell exactly how much liquid it contains, but we may run out any required volume of liquid, the volume being indicated by the *difference* between the readings before and after running the liquid out. The liquid should not be used below the bottom division.

In reading off the volume of a liquid it will be seen that the surface of the liquid is not quite flat, but curved, slightly hollow or concave. The lowest part of the curve should always be taken as the true level. In the diagram the true reading is 99 c.c. and not 100 c.c.

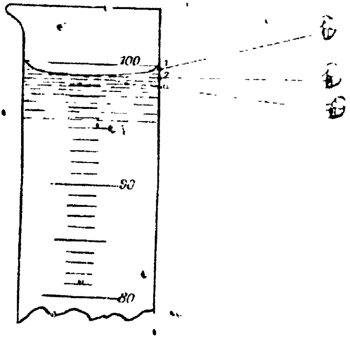


FIG. 14.

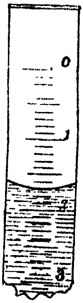


FIG. 15.

It should be observed that the eye ought to be on a level with the surface of the liquid (as at 2), otherwise the reading will be too high (as at 1), or too low (as at 3).

On first using a cylinder the student should carefully study the divisions to see what they represent. Thus in the figure there are 10 divisions between 90 and 100, and hence each division represents 1 c.c. In some cylinders there are only 10 divisions for every 20 c.c., and hence each one represents 2 c.c.

Since a burette is generally narrower, the divisions allow of finer reading; thus in the diagram (Fig. 15) there are 10 divisions for every c.c., and hence each division represents $\frac{1}{10}$ or ·1 c.c. Thus the reading of the liquid shown is 1·7 c.c.

Volume of Irregular Solids.—If a solid be placed under water it displaces or pushes away just its own volume of water. This gives a simple method of finding the volume of a small solid by lowering it into water in a measuring cylinder, reading the level before and after the solid is immersed. The difference in reading will, of course, be the volume of the solid. But a measuring cylinder, if wide enough to take a fair-sized solid, does not give very fine readings; and hence, for more accurate results, it is better to use a special displacement apparatus, in which the water pushed away by the solid is caused to overflow. The form shown in the diagram (which should be made of stout glass and firmly clamped to a base board so that it cannot wobble) has been found to give very satisfactory results. The liquid displaced overflows until the surface in the wide vessel is just on a level with the narrow nozzle. The displaced liquid may be measured in a burette or in a specially narrow cylinder.

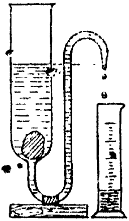


FIG. 16.

A solid lighter than water may be pushed under by a fine needle. Any air-bubbles adhering to the solid should be removed as they displace water just as much as if they were solid.

SUMMARY.

Volume of rectangular solid = length × breadth × thickness.

Volume of prism = length × area of end.

Volume of cylinder = length × $3\frac{1}{7}$ times square on radius ($l \times \pi r^2$).

Volume of sphere = $\frac{2}{3}$ vol. of corresponding cylinder ($= \frac{4}{3}\pi r^3$).

Volume of cone = $\frac{1}{3}$ vol. of corresponding cylinder ($= \frac{1}{3}l \times \pi r^2$).

Volume of irregular solids found by displacement of water.

EXERCISES.

1. A plank is 4 yards long, $\frac{3}{4}$ foot wide, and 4 inches thick. Find its volume in cubic feet and in cubic inches.

2. Find the volume of an iron girder 10 feet long and having the section and dimensions shown in Fig. 17 (p. 31).

3. What is the volume of 100 yards of wire, the diameter being $\frac{1}{10}$ inch?

4. A hollow iron pillar 8 feet high has an external diameter of 7 inches, and the metal is 1 inch thick all round. Find the number of cubic inches of metal it contains.

5. How many cubic inches are contained in a brick 9 inches long, $4\frac{1}{2}$ wide, and 3 inches thick? How many such bricks would be contained in 100 cubic yards of brickwork (making no allowance for mortar)?

6. A cylindrical tank has a diameter of 7 yards and a depth of 8 feet. How many gallons of water will it hold (remembering that a cubic foot of water weighs $62\frac{1}{2}$ lbs. and a gallon of water 10 lbs.)?

PRACTICAL EXERCISES.

1. If a number of cubical blocks (say, 1-inch cubes or 1-cm. cubes) can be obtained, build up various rectangular

solids, measure the length, breadth, and thickness, and count the cubes to confirm the formula for the volumes.

2. Take any convenient wooden box and by chalk lines show how its volume, if solid, could be cut up into cubic inches or cubic centimetres. Calculate the number of c. inches or c.c.

3. By means of a burette measure out and weigh the following volumes of water: 10 c.c., 11·5 c.c., 13·25 c.c., &c. From your results, what do you conclude to be the weight of 1 c.c. of water?

4. Weigh out 50 grams of water, and by pouring it into a measuring cylinder test the accuracy of the 50 c.c. mark.

5. Find the volume of a small bottle by finding the weight of water required to fill it. Confirm by filling it from a burette.

6. Find, in ounces, the weight of water required to fill a pint measure. From your result find the weight of a gallon of water.

7. Find the volume of a jam-pot in fractions of a pint from the weight of water required to fill it. (A spring balance with pan at the top and circular dial is convenient for such experiments.)

8. Find, by using a measuring cylinder, the number of c.c. in a pint. Hence, how many pints in a litre.

9. Find the volume of various regular and irregular solids by displacement of water, using either a graduated cylinder or some form of overflow displacement apparatus. Find the volumes of the regular solids by calculation and compare results.

10. Find the volume of a cylindrical can by calculation from the dimensions, and confirm by filling from a graduated cylinder

11. Compare the volume of corresponding cylinder, cone, and sphere, (*a*) by comparing the weight of wooden models of the same kind of wood; (*b*) by displacement of water.

CHAPTER IV

DENSITY AND SPECIFIC GRAVITY

Heavy and Light Substances.—It is well known that some substances are heavier than others (bulk for bulk). Thus a piece of iron weighs more than a piece of wood of exactly the same size, although, of course, a large piece of wood might weigh more than a small piece of iron. In comparing different substances we must therefore compare the weights of *equal* volumes.

It is convenient to compare the weights of unit volume of each substance.

Thus 1 cubic foot of water weighs 1000 ozs. or $62\frac{1}{2}$ lbs.
,, ,, ,, cast iron ,, . about 450 ,,
,, ,, ,, lead ,, . ,, 700 ,,
,, ,, ,, cork ,, . ,, 15 ,,

If we use the metric units we might tabulate the following facts:—

1 cubic centimetre of water weighs 1 gram.
,, ,, iron ,, 7·2 grams.
,, ,, lead ,, 11·5 ,,
,, ,, cork ,, 0·24 ,,

The weight of unit volume of any substance is called its density.

Thus the density of water may be expressed as $62\frac{1}{2}$ lbs. per cubic foot, or 1 gram per cubic centimetre, and that of iron as 450 lbs. per cubic foot, or 7·2 grams per cubic centimetre.

Relative Density, or Specific Gravity.—From the last set of numbers it is very easy to see the relation between water and the other substances, since the standard of weight (the gram) has been so chosen that unit volume

(1 c.c.) of water has unit weight (1 gram). Thus, since 1 c.c. of iron weighs 7·2 grams, it follows that 1 c.c. of iron weighs 7·2 times as much as 1 c.c. of water; hence also 2 c.c. of iron weighs 7·2 times as much as 2 c.c. of water; also 1 cubic foot of iron weighs 7·2 times as much as 1 cubic foot of water, and, in fact, any volume of iron weighs 7·2 times as much as the same volume of water.

This **number, 7·2, which tells how many times the substance weighs more than the same volume of water**, is called the **relative density, or specific gravity**, of the iron.

Similarly we may construct the following table of relative densities or specific gravities:—

Wrought iron	7·7	Cork	·24
Cast iron	7·2	Pine	·56
Copper	8·6	Oak	·84
Lead	11·5	Paraffin oil	·8
Mercury	13·6	Methylated spirit	·81
Gold	19·5		

It will be seen that those substances which are lighter than the same volume of water have specific gravities which are fractions less than 1.

It may be mentioned that the specific gravities of such substances as paraffin and spirit which vary in composition are not always the same, and even metals, especially different kinds of iron, vary somewhat in different samples. These relative densities are often useful in calculating the weight of a given volume of any particular substance. The point to remember is that these numbers apply equally to a large mass or to a small fragment of the substance. Thus, suppose we wish to know the weight of a block of stone 4 feet long, 1 foot wide, and 6 inches thick, we could first find its volume to be $4 \times 1 \times \frac{1}{2} = 2$ cubic feet. Now we know that 2 cubic feet of water would weigh $2 \times 62\frac{1}{2} = 125$ lbs.

If we further know that the relative density of this kind of stone is 2·4, it is clear that the stone block must weigh 2·4 times as much as the same volume of water, *i.e.* 125 lbs. × 2·4 = 540 lbs. or $\frac{540}{112} = 4·82$ cwts.

To take another example; since we know that 1 gallon of water weighs 10 lbs., it is clear that 1 gallon of paraffin oil of specific gravity ·8 will weigh 10 lbs. × ·8 = 8 lbs.

Suppose, then, that we have a cask which, when empty, weighs 58 lbs., and when full of paraffin weighs 458 lbs.; it follows that the paraffin alone weighs 458 − 58 = 400 lbs.; and since 1 gallon of it weighs 8 lbs., there must be $\frac{400}{8} = 50$ gallons in the cask.

The working might be set out more concisely :—

Weight of paraffin + cask	458 lbs.
" cask	58 lbs.
" paraffin	400 lbs.

1 gallon of paraffin weighs 10 lbs. × ·8 = 8 lbs.

Volume of paraffin = $\frac{400}{8}$ = 50 gallons.

Similar calculations might, of course, be made from the actual densities instead of relative densities. Thus, if we were required to find the weight of an iron rail 16 square inches in cross section and 36 feet long, we should first find its cubical contents or volume and then multiply by the density or weight per cubic foot.

Volume of iron = 36 feet × $\frac{}{144}$ sq. feet = 4 cubic feet.

But 1 cubic foot of wrought iron weighs 480 lbs.

∴ Total weight = 480 × 4 = 1920 lbs. or $\frac{1920}{112} = 17·1$ cwts.

EXERCISES.

1. Find the weight of a pine log 12 feet long × 15 inches wide × 12 inches thick, given that the density of pine is 35 lbs. per cubic foot.

2. Work the above question, given that the specific gravity of pine is ·56.

3. Find the weight of a sheet of lead 10 feet long, 2 feet wide, and ⅛ inch thick, by first working from the density in lbs. per cubic foot and then from the specific gravity of lead (taken from the tables).

4. What will be the thickness, in inches, of lead which weighs 7 lbs. to the square foot?

5. What will be the weight per foot run of a wrought iron girder of the section shown (Fig. 17)?

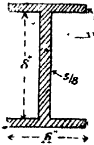


FIG. 17.

Methods of finding Specific Gravity.

As the specific gravity of a substance is simply a number which tells how many times any given amount of the substance weighs more or less than *the same volume* of water, all we have to do in finding it is to weigh a convenient quantity of the substance and then find the weight of an equal bulk of water, and see how many times the substance is heavier than the water by dividing the weight of substance by the weight of water, thus:—

$$\text{Specific gravity} = \frac{\text{weight of substance}}{\text{weight of equal bulk of water}}.$$

In the case of a liquid the process is very simple, for we may find what weight of the substance just fills a certain bottle, and then find the weight of water required to fill the same bottle. The bottle must be quite full, and no more, in both cases, hence it is convenient to use a bottle with a glass stopper in which a narrow groove has been filled, so that, when the stopper is put in, the excess of liquid is pushed out through the groove; the outside of the bottle must then be dried. Care must be taken not to enclose any air bubbles. As the liquids are weighed *with* the bottle, the weight of the empty bottle must of course be subtracted in each case.

Example :—

Weight of bottle + liquid	= 75·99	Weight of bottle + water	= 85·85
Weight of empty bottle	= 35·84	Weight of bottle	= 35·84
∴ Weight of liquid	= 40·15	∴ Weight of water	= 50·01

Hence specific gravity of liquid $= \frac{40 \cdot 15}{50 \cdot 01} = \cdot 803$.

This means that the liquid weighs only ·803 as much (*i.e.* about $\frac{4}{5}$ as much) as an equal volume of water.

In the case of a regular solid, *e.g.* a cube or a cylinder, we might find its volume by calculation from the dimensions; and then, remembering that every c.c. of water weighs a gram, we can at once write down the weight of the same volume of water, and if we also weigh the solid we may calculate the specific gravity as before.

Example :—

A copper cylinder 2 centimetres long and 1·4 c.m. diameter weighs 264·88 grams. Find the specific gravity of the copper.

$$\text{The volume of a cylinder} = l \times \pi r^2$$
$$= 2 \times 3\tfrac{1}{7} \times \cdot 7 \times \cdot 7$$
$$= 2 \times \tfrac{22}{7} \times \cdot 7 \times \cdot 7 = 30 \cdot 8 \text{ c.c.}$$

Now, 30·8 c.c. of water would weigh 30·8 grams.

$$\text{Hence } \frac{\text{weight of copper}}{\text{weight of water}} = \frac{264 \cdot 88}{30 \cdot 8} = 8 \cdot 6.$$

That is, the copper is 8·6 times heavier than an equal volume of water.

The Weight of a Solid in Air and Water.—If we have to deal with an irregular solid, its volume might be found by displacement, as described in the last chapter; but a simpler and more accurate method depends on the apparent loss of weight of a solid, when suspended in water. To understand this loss of weight, let us consider the following experiment. If we have two tall vessels, one containing pure water, and the other containing a strong solution of salt in water, it will be found that an egg will sink in the pure water, but it will float in the salt water. We explain this, commonly, by saying that the egg is heavier than pure water, but lighter than salt water; we mean, of course, that the egg is heavier than its *own bulk* of pure water and lighter than its own bulk of strong salt water. By carefully mixing the salt water with some pure water we can gradually reduce its strength until the egg will neither sink to the bottom nor float on the top, but remains indifferently anywhere under the liquid. We should then be justified in concluding that the egg is now just as heavy as its own bulk of the weaker salt water.

Let us now consider the three cases. The strong salt water evidently pushes the egg upwards, with a force greater than the downward force due to the weight of the egg. In the case of the weaker salt water the upthrust (or upward force) just balances the weight of the egg, so that the egg apparently loses all its weight but is not forced to the top. Thus the upthrust is clearly equal to the weight of the egg, but since the egg, in this instance, is just as heavy as its own bulk of liquid it follows that the upthrust is also equal to *the weight of the same bulk of liquid*.

In the case of the pure water, although the egg falls to the bottom it does not fall nearly so quickly as in air, and it seems likely that, here also, there is an upthrust, which, however, only partly neutralises the weight of the egg. To find out whether this is the case we might suspend the egg by a fine thread so that its apparent weight might be found while hanging in the water. It will be found that there is indeed a very considerable loss in the apparent weight, or in other words, the water does exert an upthrust. Now in the last case we found that this upthrust (or apparent loss of weight) was equal to the weight of liquid just equal in volume to the egg, or in other words, just equal to the weight of the liquid displaced by the egg. To find out whether this is also the case with the pure water we might find by experiment with the displacement apparatus (Fig. 16) the volume of water displaced by the egg, and the result will be that this volume, expressed in cubic centimetres, corresponds with the loss of weight of the egg expressed in grams, and since each cubic centimetre of water weighs a gram, we conclude that **the loss of weight of the solid when suspended in water** is **just equal to the weight of its own volume of water** (or of the volume of liquid displaced). This statement is known as the **Principle of Archimedes**, and is very useful in finding the specific gravities of solids by weighing first in air and then in water and so obtaining the loss of weight (see Fig. 18).

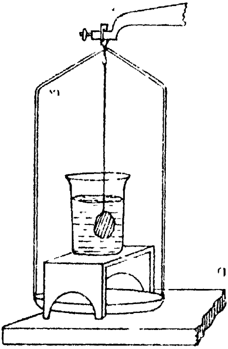


Fig. 18.

Example (compare with last example, p. 32):—

Weight of copper cylinder in air = 264·88 grams.
Weight of same in water = 234·08 „

Loss of weight = weight of equal vol. water = 30·8 „

Hence sp. gravity $= \dfrac{\text{wt. of copper}}{\text{wt. of water}} = \dfrac{264{\cdot}88}{30{\cdot}8} = 8{\cdot}6.$

In the case of a solid in powder or in small pieces, the weight of an equal volume of water can be found by weighing a specific gravity bottle full of water, and then finding the weight of water pushed out when the solid is put into the bottle.

Example :—

Weight of lead shot . . . . = 100 grams.
Weight of bottle full of water . = 85·85 ,,
Total weight . . . = 185·85 ,,
Weight of bottle + water + shot (inside) = 177·16 ,,
Weight of water displaced by shot = 8·69 ,,

Hence sp. gravity $= \frac{\text{wt. of lead}}{\text{wt. of water}} = \frac{100}{86·9} = 11·5$

SUMMARY.

Density = weight of unit volume (*e.g.* in lbs. per cubic foot or grams per c.c.). If density is expressed in grams per c.c. it indicates how many times the substance is heavier than water (since 1 c.c. of water weighs 1 gram).

Specific gravity or relative density $= \frac{\text{weight of substance}}{\text{weight of } \textit{same vol.} \text{ of water}}$:

it tells how many times the substance is heavier or lighter than water (bulk for bulk).

Weight in *grams* of given volume of substance = number of *c.c.* × sp. gravity.

Volume in c.c. of given weight of substance = number of grams ÷ sp. gravity.

Principle of Archimedes: when a solid is immersed in liquid, the loss of weight = weight of liquid displaced.

Specific gravity of solid $= \frac{\text{weight of solid in air}}{\text{loss of weight in water}}$.

EXERCISES.

1. A gallon of oil is found to weigh 8½ lbs. What is its specific gravity?

2. A bottle weighs 25 grams when empty, 57 grams when full of water, and 52·8 grams when full of turpentine. Find the specific gravity of the turpentine.

3. What would be the weight of a gallon of the above turpentine?

4. A metal ball weighs 25·3 grams in air and 23·1 grams in water. Find the specific gravity of the metal.

5. An oak plank (sp. grav. ·8) 3 inches thick floats on water. What thickness of it will stand above water?

PRACTICAL EXERCISES.

1. Determine by measurement the volume of various regular solids of wood and metal, then weigh them, and calculate the densities in grams per cubic centimetre and in lbs. per cubic foot.

2. Find the volume of a small bottle by the weight of water required to fill it. Find the weight of various other liquids (*e.g.* methylated spirit, salt solution, &c.) required to fill it, and hence calculate the densities of the liquids, in grams per c.c., and also their specific gravities.

3. Find the volumes of various irregular solids (*e.g.* bottle stoppers, pieces of lead pipe, metal cylinders, &c.) by displacement; then weigh them and find their densities and specific gravities.

4. Weigh the above solids in air and water; compare the losses in weight with the weight of an equal volume of water (as found in experiment 3), and again find the specific gravities.

5 Find the specific gravity of lead shot (or iron tacks, &c.) by weighing alone and then in a bottle previously filled with water and weighed, and from the weight of water pushed out find the specific gravity of the solid.

6. Find the specific gravity of a solid lighter than water (*e.g.* a waxed cork) by weighing in water, using a sinker of known weight and density.

CHAPTER

LEVERS

We all know that, on a pair of scales with equal arms, two equal weights will just balance each other; but it is also a familiar fact that on a common see-saw it is possible for a light person to balance a heavier one by sitting farther away from the support on which the see-saw swings. This introduces the idea of leverage.

Experiments can be made with a wooden rod arranged to swing on a pivot, the weights being hung by loops of string, so that they can be moved along the rod to different distances from the pivot or **fulcrum**. A 1 lb. weight 2 feet from the fulcrum will balance 2 lbs. placed 1 foot from the fulcrum, and if the 1 lb. weight be moved to a distance of 3 feet, it will then balance 3 lbs. at 1 foot distance or 2 lbs. at 1½ feet distance.

Thus in each case the *number* of pounds on one side multiplied by the *number* of feet in the distance from the fulcrum will be just equal to the corresponding product on the other side.

We might tabulate the results as follows:—

Left Side.			Right Side.		
Weight.	Distance.	Product Weight × Dist.	Weight.	Distance.	Product Weight × Dist.
1	2	2	2	1	2
1	3	3	3	1	3
1	3	3	2	1½	3
2	3½	7	7	1	7

Other experimental results may be added by the student.

From these results it appears that a small weight (or force) acting at a great distance from the fulcrum will support a larger weight at a short distance. It is convenient to regard one of the weights as the *power* which balances the other *weight* or *resistance*; and we may then state the following rule or *law* of the lever, viz.—

Power × its distance from fulcrum = resistance × its distance from fulcrum, or $\frac{\text{resistance}}{\text{power}} = \frac{\text{distance of power}}{\text{distance of resistance}}$.

Thus the lever enables us to multiply the effective force many times; for instance, if the power be applied 5 times as far away as the weight, the power required will only be $\frac{1}{5}$ of the weight or resistance.

The relation between the weight and the power in any machine is called the *mechanical advantage*. Thus in the above example the mechanical advantage is 5.

We may state the relations as follows:—

$$\text{Mechanical advantage} = \frac{\text{resistance}}{\text{power}} = \frac{\text{distance of power}}{\text{distance of resistance}}.$$

It is not necessary that the fulcrum should be a fixed pivot; all that is required is a steady support upon which the bar can swing, as in the case of a common crow-bar used in raising a heavy weight (see Fig. 19). It should be clear that to make the mechanical advantage as great as possible, the fulcrum should be placed very near to the weight to be raised, and the power should be applied as far away as

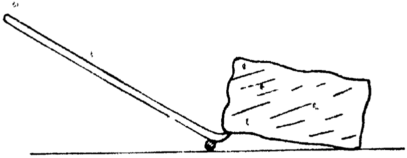


Fig. 19.

possible. Thus, if the power be applied 20 times as far away as the weight, the power required will be $\frac{1}{20}$ of the weight, whereas if the power be applied at 40 times the distance it need only be $\frac{1}{40}$ of the weight; in fact, the weight and power are *inversely proportional* to their respective distances from the fulcrum.

Example :—

Suppose a girder weighing 5 tons is to be raised by four crow-bars (two at each end), the weight being equally divided between them, and imagine also that the fulcrum is placed in each case 3 inches from the girder and the power applied 5 feet from the fulcrum. Find what power (expressed in cwts.) must be applied to each crow-bar.

The mechanical advantage will be

$$\frac{5 \text{ feet}}{3 \text{ inches}} = \frac{60 \text{ inches}}{3 \text{ inches}} = 20.$$

Hence the power required will be $\frac{1}{20}$ of the weight.

But the weight to be raised by each bar will be $\frac{1}{4}$ of 5 tons, *i.e.* $\frac{1}{4}$ of 100 cwts. = 25 cwts.

Hence the power necessary will be $\frac{1}{20}$ of 25 cwts. = $1\frac{1}{4}$ cwt.

In reality the weight of the crow-bar itself will assist the power applied, but this has been left out of account for simplicity.

This simple appliance is thus a very effective means of obtaining increased lifting power, but we must not forget that there is a corresponding disadvantage. Thus in the diagram, since the power is applied at P, four times as far from the fulcrum, F, as the weight W, the mechanical advantage is 4, *i.e.* P will be only $\frac{1}{4}$ of W. But it is easily seen that the distance WW' (which may be called the range of motion), through which the weight is

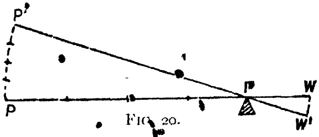


FIG. 20.

raised is only ¼ of the distance PP′ through which the power has been exerted, thus while the power is multiplied fourfold the distance is diminished fourfold, or what has been gained in power has been lost in the distance covered.

This is a very important principle which holds good in all "machines" or mechanical appliances for the multiplication of power. It may be called the **Principle of Equal Work,** and may be expressed by stating that the work done by a machine is always equal (if we neglect losses by friction) to the work done upon it. Thus we may use a machine to multiply *force* but we cannot use it to multiply *work*. (See next chapter.)

Other Types of Levers.—With our pivoted wooden rod it is possible to suspend the weight between the fulcrum and the power. In order to raise the weight it will then be necessary for the power to be directed upwards. This may conveniently be done by means of a spring balance, or by attaching a weight to a string passing over

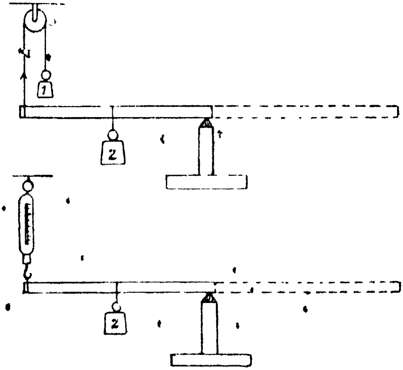


FIG. 21.

a pulley. The right-hand part of the lever, dotted in the diagram, then becomes unnecessary, but it is convenient to retain it in order to neutralise the effect of the weight of the bar.

The student should make and tabulate a series of experiments to find out whether the same law applies in this "lever of the second order" as the one which held good in the first order.

It will again be found that the product of the power and its distance from the *fulcrum* equals the product of the weight and its distance from the fulcrum; and the mechanical advantage will be found in the same way as before.

There is still another possible arrangement constituting the third order of levers in which the power is applied at a point between the fulcrum and the weight. In this case it will be found that the upward pull of the power will tend to raise the lever, and hence the fulcrum must bear upon the top of the lever instead of below, and it will not be possible to neutralise the weight by a second arm of the bar. Allowance is, however, easily made for this weight by taking a reading of the spring balance (or by balancing the weight of the bar with shot, if a pulley is used) before the weight is attached. It will then be possible to prove that the same general law still holds good, *i.e.* power × distance = weight × distance.

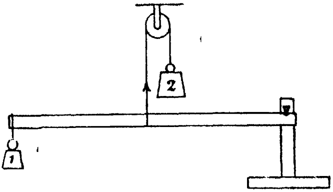


FIG. 2[illegible]

It will be observed that in this kind of lever, since the power is always nearer to the fulcrum than the weight, the power must always be greater than the weight, *i.e.* the mechanical advantage (as it is still called) will be a fraction. Thus, if the power be applied half-way between fulcrum and weight, the mechanical advantage will be ½. But, on the other hand, in order to raise the weight 2 inches, it will only be necessary for the power to be applied through 1 inch. Levers of this kind are sometimes used in order to secure a greater range of motion. An excellent example is afforded by the human arm. The bones of the forearm may be considered as a lever, with the fulcrum at the elbow. The muscle called the biceps, situated in the upper part of the arm, is attached to the bone of the forearm a *short* distance from the elbow, and hence a slight contraction of this muscle will cause the hand at the other end of the lever to move through a much greater distance

Moment of a Force.—The weight on a lever tends to turn it about the pivot or fulcrum in one direction (let us suppose it to be "clockwise," *i.e.* the way the clock-hands move), and the power tends to turn it in the opposite direction ("anti-clockwise"). It has been seen that the effect of the power (or of the weight) increases the further it is applied from the fulcrum, and, in fact, the turning effect is measured by the product of the force by its distance from the fulcrum. This product, which measures the turning power of the force is called the "moment" of the force, and the law of levers may be stated thus: the moment of the power = the moment of weight or resistance.

Applications of Levers.—Levers may be used for the following purposes:—

1. To obtain increased power.
2. To increase or decrease the range of motion, or to transmit force to inaccessible places.
3. To compare weights (as in a steelyard) or to balance forces (as in the lever steam safety valve).

The first two uses are exemplified in the various brake and signal levers.

Many levers are bent; for instance, the lever for applying the brake on a railway waggon. The ordinary rule for finding the mechanical advantage generally applies in these bent levers, but in some cases it requires modification.[1]

A common pump-handle, the handle of a smith's bellows, the handle of a mortising machine, and a coachman's lifting jack are levers used to obtain increased power.

The "picking-peg" of a loom, which drives the shuttle across the warp, is a lever of the third order, used to obtain increased range of motion.

The oar of a boat is a lever of the *second* order, since the resistance of the boat acts through the rowlocks and the water forms the fulcrum.

Many simple appliances consist of a pair of levers with a pivot to act as fulcrum, *e.g.* shears, pliers, pincers, tongs, and nut-crackers. The student should consider in each case which order of lever is represented.

In many cases the principle of leverage comes in incidentally. Thus, in shutting a door or gate the power is more effectually employed the further it is applied from the hinges, which form a fulcrum. A nut is easily cracked by squeezing it between a door and the doorpost. This method of cracking nuts is not recommended, however, as it places a great strain on the hinges, for if we consider the nut to act as a fulcrum, the leverage will tend to wrench off the hinges.

A claw-hammer used to extract a nail forms a bent lever of the first order. The metal rod carrying the ball of a ball-tap, for supplying water cisterns, is also a bent lever of the first order.

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[1] The point to be remembered is that, whatever be the shape of the lever, the arms are to be measured by taking the direct perpendicular distances between the fulcrum and the lines of action of the forces concerned.

the back of a chair; the cycle frame is hung over one side just 1 foot from the chair-back, and it is found to be just balanced by a 7-lb. weight 5 feet from the fulcrum. What is the weight of the bicycle?

4. In a steelyard the object is suspended 2 inches from the fulcrum and the sliding weight amounts to 4 lbs. How far must this be placed from the fulcrum to balance ½ cwt.? What must be the weight of an object which is just balanced at a distance of 18 inches? How far apart must the marks on the scale be arranged for each division to correspond to ½ lb.?

5. In a safety valve the effective aperture through which the steam escapes has an area of a square inch. If the valve lever has its fulcrum 2 inches from the valve, and carries a weight of 5 lbs., how far must the weight be placed from the fulcrum so that the steam may blow off at a pressure of 80 lbs. on every square inch? What would be the blow-off pressure if the weight were placed 1 foot from the fulcrum?

PRACTICAL EXERCISES.

1. Tabulate various weights and distances in levers of the three orders in the manner indicated in the text.

2. Using a lever of the first order with, say, 50 grams on one side and ½ lb. on the other (or any convenient sizes), find (*a*) how many lbs. are contained in a kilogram, and (*b*) how many grams in an ounce.

3. Use a lever as a steelyard to find the weight of a bag of sand or other object. Place the object at various distances, and compare your results.

4. Using a lever of the first order, place two or more weights at different distances on one arm, and balance by a single weight on the other arm. Does the sum of the "moments" (distance × weight) on one side equal the single moment on the other side?

5. If possible, use a prop and chair-back to weigh a bicycle in the manner indicated in question 3 above. It will be convenient to make a small nick at the middle of the prop, so that it may be readily poised on the sharp edge of the chair-back.

CHAPTER VI

FORCE, WORK, AND ENERGY

Force.—If we wish to raise a weight we have to exert "force." This is because the weight is acted upon by gravitation, which is another force causing the weight to be attracted towards the earth; and, in order to overcome this attraction or downward pull, we must exert a stronger upward pull.

If a body be unsupported, the force of gravitation sets it in motion, and causes it to fall towards the earth. A body resting on the ground is still acted upon by the force of gravitation, but this force is opposed and resisted by the upward pressure or reaction of the ground.

Thus it appears that a force may tend to produce motion, or it may, by opposing another force, arrest motion, or it may arrest the motion of a body which is already moving (*e.g.* the brake on a train), or it may change the direction of its motion (*e.g.* when a cricket-bat strikes the moving ball). When two opposing forces just balance each other they are said to be in **Equilibrium**.

Mass and Weight.—Since the force of gravitation is a universal, and, so long as we remain in one place, a uniform force, it is often convenient to use it in comparing and measuring other forces.

Thus, in the last chapter, in dealing with the lever we used weights as measurable forces.

The student should be careful to note that the unit called the pound may be used in two quite different senses: thus, if we are considering *the quantity of material* in it, it is a measure of **mass**: whereas if we are considering the *force* with which it is attracted to the earth, it is a measure of **weight**.

For example, if we are *carrying* a parcel we are concerned with its weight—the heavier it is the harder it will be to carry; but if we are considering the *contents* of the parcel we are concerned with its mass—the greater the mass the more material we have. If we imagine the parcel removed to the surface of the moon (where the force of gravitation is about six times weaker than on the earth), its weight would be reduced to about one-sixth of its former amount, but the mass would remain as before.

Again, if we imagine a lump of iron weighing 1 lb. to be hung from a spring balance, the latter should record a weight of 1 lb.; but if we bring a powerful magnet under the lump of iron it will appear to become heavier, or its apparent weight will increase because the pull of the magnet is added to the pull of the earth, but, of course, the mass or quantity of iron is just the same as before.

It happens that, so long as we remain in one place, where the force of gravitation is uniform, the weights of bodies are proportional to their masses (whatever be the kind of material), and hence weight is often used as a convenient *measure* of mass, although the two things are really quite different.

Work.—It will now be convenient to consider a method of measuring work. Suppose a labourer is carrying bricks or mortar to the top of a building, it is clear that the work he performs on each journey will depend upon two things, viz. (1) the weight he carries, and (2) the height to which he carries it. Thus, if he carries twice as much for the same distance, or if he carries the same weight for twice the distance, in both cases the work done will be doubled. If he carries twice as much for three times the distance, he will do $2 \times 3 = 6$ times as much work. Hence in measuring the work done in raising a weight we must introduce both the unit of weight (or force) and the unit of distance. In this country the commonest unit of work is called the **foot-pound,** which represents the work done in steadily raising a weight of 1 lb. *vertically* upwards through a distance of 1 foot. The number of foot-pounds of work done in raising a weight will be the product of the number of pounds by the number of feet through which the weight is raised. Thus, to raise 5 lbs. through a vertical height of 6 feet we must do $5 \times 6 = 30$ foot-pounds of work.

This should make it clear why a lever does not multiply work; because while it may multiply the power (so as to raise a heavier weight), it diminishes the distance just as many times; and hence the product of the two, which is the measure of the work, remains unchanged.

It may be observed that we may put forth great strength in the endeavour to move a heavy weight, but if we do not succeed in moving it no work will have been accomplished in the scientific sense; the work done depends, in fact, on the result produced, and not merely upon the effort put forth.

It may be added that the foot-pound (or the "foot-ton") may be used in measuring other kinds of work than that done in raising a weight. Work may be defined as the overcoming of resistance, and wherever that resistance can be measured in pounds (or tons, &c.) the work will be measured by multiplying the number of pounds (or tons) by the distance in feet through which it is overcome.

For example, if an engine pulling a train on the level at a given speed requires (in order to overcome the frictional resistances) to exert a pull which would just suffice to raise a weight of 5 tons vertically upwards, then the engine does 5 foot-tons of work for every foot through which it moves the train on the level.

If the train were travelling uphill, we should have to add to the work which would be done on the level, the work done in raising the train vertically up. Thus, for instance, if the train is going up an incline of 1 in 100, it would rise vertically 1 foot for every 100 feet the train travels on the track: and supposing the weight of the train to be 200 tons, the *extra* work done in travelling 100 feet would be 200 foot-tons, or 2 foot-tons for every foot on the track.

To take another kind of example, let us suppose that in sawing wood the average force exerted in the thrust of the saw would be just sufficient, if directed upwards, to raise a weight of 10 lbs., and suppose the length of the thrust to be 18 inches, then the work done during each thrust would be $10 \times 1\frac{1}{2} = 15$ foot-pounds.

Energy.—In order to do work we must possess a store of what is called "energy." This energy can be measured in foot-pounds according to the amount of work it can do.

For example, in a pile-driver a heavy block of iron is raised by the men and allowed to fall upon the pile. In driving in the pile against the resistance which it offers, work is done; this work can be measured by the weight of the block and the height from which it falls. Thus, if it weighs 5 cwts. (*i.e.* $\frac{5}{20}$ ton) and falls 8 feet, then $\frac{5}{20} \times 8 = 2$ foot-tons will represent the work done at each stroke.

When the block is at the top it possesses a store of passive energy in virtue of its raised position, and when i is in the act of falling this is changed into active energ due to its motion. The first or passive form is calle **potential energy**, the second or active form is call **kinetic energy** or energy of motion.

By way of example we may mention a falling hamn a moving cannon-ball, a revolving flywheel, a movi train, falling water, wind (or moving air), as cases of kine energy; whereas the raised weights of a clock, the coiled spring of a watch, a store of water at a raised level (*i.e.* a "head" of water), are examples of potential energy which can be employed for useful work.

But there are other ways in which energy (or working capacity) can be stored. Thus a piece of coal represents stored-up energy; for by burning it and generating steam we can drive a steam-engine. Gunpowder and dynamite contain vast stores of energy, which are suddenly liberated when an explosion occurs, and work may be done either in propelling a bullet or in rending a rock.

The Law of Conservation of Energy.—The various forms of energy are capable of transformation one into another, *e.g.* the potential energy of the raised pile-driver is converted into kinetic energy as it falls. The chemical energy of coal is converted into heat energy when it burns, and the latter into mechanical energy by the engine. Many attempts have been made to invent a machine which in some way would drive itself and yield perpetual motion. Such attempts have always failed, and always must fail, simply because it is a fundamental law of nature that energy can neither be created nor destroyed. This is simply another and more general way of stating our principle of equal work (mentioned in the previous chapter).

It should be understood that although energy cannot be destroyed it may be wasted as far as any useful application goes. Thus a machine practically always turns out less work than we put into it, on account of losses due to friction. This friction gives rise to heat, which is, of course, a form of energy; but it is a form which is not available for useful application, hence, although the *total* energy remains the same, the *useful* energy is reduced by friction. One object in designing a machine should be to reduce this waste of energy to a minimum.

SUMMARY.

Force may produce motion, or arrest it, or change its direction.

When forces balance each other they are in equilibrium and no motion results.

Mass is the quantity of material in a body.

Weight is the downward force due to the pull of gravity on a body.

The **Foot-pound** is the unit of work = work done in raising 1 lb. through 1 foot (vertically).

Energy is the power of doing work; *i.e.* overcoming resistance.

Potential energy is the passive form; *i.e.* energy of position.

Kinetic energy is the active form; *i.e.* energy of motion.

Conservation of Energy: energy cannot be created or destroyed—hence we cannot get more work out of a machine than is put into it.

But energy may be wasted; *e.g.* by friction.

EXERCISES.

1. How many foot-pounds of work will be done in bringing a ton of coal from the bottom of a mine 500 feet deep?

2. A pile-driver weighing 1 cwt. is raised 8 feet before each stroke, and 50 strokes are necessary to drive in a pi
How many foot-tons of work are expended in the process

3. A water-wheel uses 1000 gallons of water per minute and the water falls 10 feet in the process. Taking a gallon of water as 10 lbs., how many foot-tons of work will the wheel accomplish in an hour?

If we take 33,000 foot-pounds per minute as being equivalent to 1 horse-power, what will be the horse-power of the water-wheel?

4. How much work must be done in pumping 1000 gallons of water from the bottom of a well 300 feet deep, adding on 10 per cent. (*i.e.* 10 parts in 100 or 1 in 10) for the friction of the pump?

5. In punching rivet-holes in a ½ inch plate the average pressure of the punch is 5 tons. Find in foot-pounds the work done in punching each hole.

6. A train weighing 150 tons is going up an incline of 1 in 200. How many foot-tons of energy will be consumed in lifting the train for every mile travelled?

7. An engine running on a level track exerts a pull of 1½ tons. How much energy will be consumed every minute if the train is going 20 miles an hour?

CHAPTER VII

THE WHEEL AND AXLE

Belt Pulleys and Toothed Wheels

THE lever, which gives us command of greatly increased power, suffers from the disadvantage that its range of motion is very limited; for instance, the distance through which a weight can be raised by a single application of a crow-bar is very small.

This disadvantage is overcome in the wheel and axle, which may be considered as a kind of revolving lever. The form of this machine, shown in the diagram, consists of a grooved wheel attached to a thick axle or roller, so that the two revolve together on the same spindle. The power is applied to a cord passing round the groove, and for experimental purposes this power may take the form of a weight hung on the cord. The load to be raised is suspended from a cord wound in the other direction round the axle so that when the smaller weight attached to the wheel descends, the cord carrying the load is wound up on to the axle and so the load is raised. The small diagram shows an end-view of the arrangement, and the dotted line Rr may be regarded as a lever of the first order, whose fulcrum is formed by the spindle.

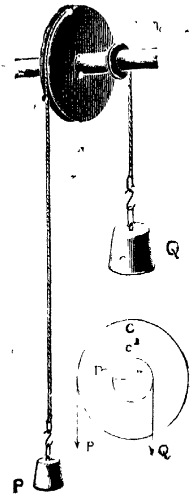

FIG. 25.

Let us suppose that the radius of the wheel is three times the radius of the axle, then clearly the arms of the lever will be in the ratio of 3 : 1 and the mechanical advantage will be 3, or the power P will balance a load Q just 3 times as great.

As the wheel and axle revolve it will always be possible to draw horizontal lines similar to Rr to form an imaginary lever, and hence the same leverage and the same mechanical advantage will be maintained, so that the only limit to its range will be the length of rope.

Consider now the relation between the distances travelled by the power P and the load Q. If P falls through a height equal to the circumference of the wheel, it will clearly cause the wheel (and with it the axle) to turn just once round. As the axle turns once, it will wind up the weight through a height equal to the circumference of the axle; and, since the radius of the axle is just one-third of the radius of the wheel, it follows that the circumference of the axle will also be one-third of the circumference of the wheel, and hence the distance through which Q rises will be just one-third of that through which P falls. Thus we find that, while the force P raises a load Q three times as great, it only raises it one-third the distance through which P falls; i.e. the principle of equal work applies here just as in the case of levers: what is gained in power being lost in the distance covered; or, in other words, the work done by P in falling is just equal to the work done on Q when it is raised.

From the above example it will be clear that the mechanical advantage of any wheel and axle will be found by dividing the diameter of the wheel by the diameter of the axle, or the circumference of the wheel by that of the axle.

$$\text{Mechanical advantage} = \frac{\text{diameter of wheel}}{\text{diameter of axle}}.$$

$$\text{Speed ratio} = \frac{\text{distance covered by load}}{\text{distance covered by power}} = \frac{\text{diameter of axle}}{\text{diameter of wheel}}.$$

Instead of applying the force by a weight at the circumference of a wheel, it is more usual in practice to have a handle attached to a crank as in a common draw-well or windlass. As the handle revolves it describes a circle, which corresponds to the circumference of the wheel in the other form of apparatus.

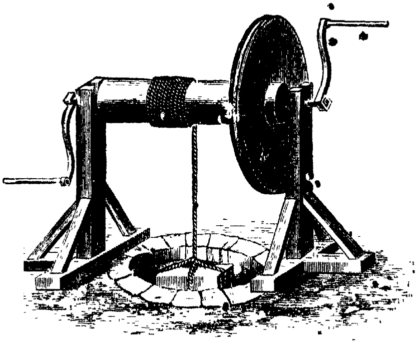

FIG. 20

The capstan, used on boat or ship for raising the anchor, &c., consists of a vertical cylinder round which the rope or chain is wound, and it is rotated by means of a number of handspikes acting like revolving levers.

In many cases it is more convenient, instead of using a wheel and axle attached to the same spindle, to use two wheels revolving on different spindles and geared together by means of teeth or cogs, or two pulleys arranged so that one rotates the other by means of a belt, or, as in bicycle gearing, two toothed wheels connected by a chain.

All these arrangements depend on exactly the same fundamental principle as the wheel and axle. It will be found that the mechanical advantage is most readily arrived at from the distance ratio or speed ratio of the different wheels.

Belt Pulleys.—The diagram represents two pulleys A and B connected by a belt. Let us suppose that B has twice the diameter of A (and therefore twice its circumference), and that they are keyed to equal axles.

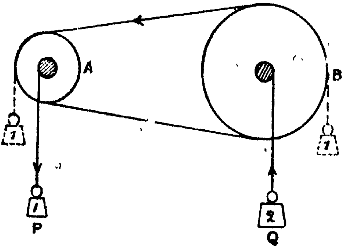

FIG. 27.

If A revolves once the belt will travel forward by a distance equal to the circumference of A and will cause B to revolve, but since B has double the circumference of A, it will only make half a turn for every turn of A.

Suppose that A is driven by a falling weight P, attached to its axle, and causes the load Q, attached to the other axle, to rise, it is clear that since the axles are equal, and since B only makes half as many turns as A, that Q will rise only half as far as P falls, and hence by the principle of equal work if P and Q are adjusted to balance each other Q must be just twice as heavy as P. For instance, if P is 1 lb. and falls 1 foot, it will do 1 foot-pound of work, and since Q only rises ½ foot, it must weigh 2 lbs. in order that 1 foot-pound of work may be done on it. Thus we get the result that when A drives B the mechanical advantage is 2 and the speed ratio ½.

On the other hand, if B drives A the mechanical advantage will be ½ and the speed ratio 2. Thus it appears that by driving a large pulley by a smaller one there is a gain in power with corresponding loss of speed; whereas if we use a large pulley to drive a small one there is gain of speed and loss of power. It should be carefully observed that if the weights P and Q were attached to the circumference of the pulley (as shown in dots in the diagram) the weights would have to be *equal* in order to balance, since the pull on the belt is the same throughout (neglecting friction). The mechanical advantage thus only applies to the driving effect *at the axles*.

From the above example it will be clear that the speed ratio and mechanical advantage depend on the ratio of the diameters of the two pulleys.

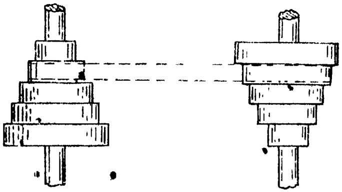

FIG. 28.

Most lathes are provided with speed-cones, each consisting of several pulleys, arranged step-wise, so that different speed-ratios can be obtained by moving the belt from one pair of pulleys to another, the steps being so adjusted that the same length of belt will fit any pair.

Belting and pulleys are frequently used to transmit power from one place to another quite apart from any mechanical advantage. It will be observed that with a direct belt both pulleys must revolve in the same direction, but if the belt be crossed the pulleys will revolve in opposite directions.

Toothed Wheels.—Wheels are frequently geared to work together by means of teeth or cogs. The mechanical advantage and speed ratio can be found from the radii or diameters of the wheels, just as in the case of pulleys, but it is often simpler to count the teeth. Since the distance between two teeth (called the pitch of the teeth) must be the same in both wheels in order that they may fit together, it follows that the numbers of teeth must be proportional to the circumferences, and therefore also to the diameters of the wheels; thus if one wheel has twice the diameter of the other, it will clearly have twice as many teeth.

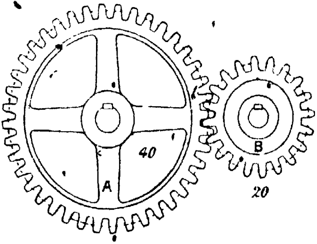

FIG. 20

If a wheel A with 40 teeth gears with a wheel B having 20 teeth, B will revolve twice for every time that A revolves once, and if A drives B, the speed ratio will be 2 and the mechanical advantage $\frac{1}{2}$.

The wheel which drives the other is called the driver, and the other is termed the follower or driven wheel.

We may therefore state the following relations:—

$$\text{Speed ratio} = \frac{\text{Revolutions of follower}}{\text{Revolutions of driver}} = \frac{\text{number of teeth on driver}}{\text{number of teeth on follower}}.$$

or

$$\text{Mechanical advantage} = \frac{\text{number of teeth on follower}}{\text{number of teeth on driver}}.$$

It will be observed that toothed wheels in direct gear revolve in opposite directions. If it is desired that the two wheels shall turn the same way an intermediate "idle" wheel may be placed between them (C in Fig. 29A).

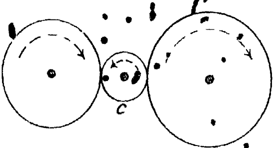

Fig. 29A.

The number of teeth on the idle wheel will not affect the speed ratio nor the mechanical advantage.

Trains of Wheels.—If a large speed ratio or a large mechanical advantage are required, it is more convenient to obtain it by successive steps by using a train of wheels. Such a train is represented in the diagram. It will be observed that two toothed wheels are keyed on to each of the middle axles, one acting as a follower to the previous axle, and the other as a driver of the following axle. Thus, if power is taken from A, then A, C, and E are drivers, and B, D, and F followers. The speed ratio is then found by multiplying the teeth of all the drivers and dividing by the product of the number of teeth of the followers.

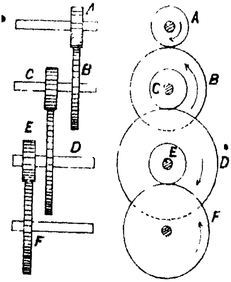

Fig. 30.

Thus, speed ratio of F to A $= \frac{A \times C \times E}{B \times D \times F}$ where the letters represent the number of teeth on the respective wheels.

Applications.—The diagram, which represents a simple "winch" or "crab," illustrates a very useful combination of the wheel and axle and toothed-wheel gearing.

It will be seen that the crank handles are attached to a shaft carrying a small pinion-wheel which gears with a much larger spur-wheel attached to the drum or barrel.

Let us determine the mechanical advantage by applying the principle of equal work.

Suppose the diameter of barrel = 8 inches.

Length of cranks = 12 inches.

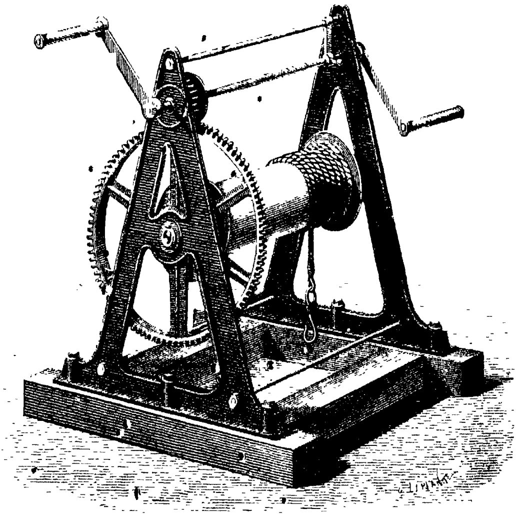

FIG. 31.

Hence the diameter of circles described by the handles 24 inches.

Number of teeth on pinion or driver, 12.

Number of teeth on spur-wheel or follower, 60.

Since the pinion has only one-fifth as many teeth as the spur-wheel, it follows that the handles must revolve *five* times in order to turn the spur-wheel, and with it the barrel, once round.

Now the diameter of the circle described by the handles is 3 times the diameter of the barrel, and hence also the circumference of the first circle (*i.e.* the distance actually travelled by the handle in one complete turn) will be 3 times the circumference of the barrel (*i.e.* the distance the rope will be wound up in one turn).

Therefore the 5 turns of the handle (required for one turn of the barrel) will carry the handles through $5 \times 3 = 15$ times the distance through which the rope is wound up. Hence the distance moved by the power is 15 times the distance moved by the weight; *i.e.* the speed ratio is $\frac{1}{15}$, and hence by the principle of equal work the mechanical advantage is 15.

This result may also be arrived at by considering the crab as a combination of the wheel and axle and the gear wheels.

If the handle drove the barrel direct, the mechanical advantage would by the usual rule be

$$\frac{\text{Diameter of wheel}}{\text{Diameter of axle}} = \frac{24}{8} = 3.$$

The mechanical advantage of the gear wheels alone would be

$$\frac{\text{Number of teeth in follower}}{\text{Number of teeth in driver}} = \frac{60}{12} = 5.$$

By combining the two we get a mechanical advantage which is the *product* of these two results; *i.e.* $3 \times 5 = 15$.

In this way the mechanical advantage of any compound machine may be found by finding the advantage for the parts separately and *multiplying* the results together. The student should particularly note that the separate results are not to be added together.

The chief uses of trains of wheels and other gearing are:—

(*a*) To obtain increased power.

(*b*) To obtain greater speed.

(*c*) To change the speed in a definite ratio.

The crab and the gearing of a hand crane are good examples of the first use.

The gearing of a bicycle and the belt and pulleys used in driving a dynamo at great speed off a shaft moving at a much slower rate are examples of the second use.

The best known example of the third use is afforded by the train of wheels in an ordinary clock or watch. The spindle carrying the hour-hand, which revolves once in 12 hours, drives that of the minute-hand once in 1 hour, and the latter drives the second-hand, revolving once a minute, the speed of the whole being controlled by the pendulum or balance wheel.

In a screw-cutting lathe the cutting tool is caused to follow the groove between the threads of the screw to be turned by means of a leading screw which carries the slide rest steadily along from left to right (or *vice versâ*). This leading screw is driven from the lathe spindle by a train of wheels so arranged that different change-wheels may be introduced in order to provide for a different rate of travel of the slide rest to adapt it to the cutting of screws of different pitch. This relation will be better understood after reading the chapter on screws.

Another important example of the use of change wheels to give definite variations of speed is afforded by various cotton-spinning machines in which the change-wheels are used to vary the "draft" and "twist" as required by different classes and counts of yarn.

SUMMARY.

$$\text{Mechanical advantage of wheel and axle} = \frac{\text{diameter of wheel}}{\text{diameter of axle}}$$

$$\text{Speed ratio} = \frac{\text{diameter of axle}}{\text{diameter of wheel}}$$

In toothed wheels and belt pulleys :—

$$\text{Speed ratio} = \frac{\text{diameter (or no. of teeth) of driver}}{\text{diameter (or no. of teeth) of follower}}$$

In trains of wheels :—

$$\text{Speed ratio} = \frac{\text{product of teeth nos. of drivers}}{\text{product of teeth nos. of followers}}$$

In any compound machine the total mechanical advantage is the product of the mechanical advantages of the separate parts.

EXERCISES.

1. In a bicycle, a gear-wheel with 45 teeth drives a chain-ring with 18 teeth attached to the back wheel. How many revolutions will the wheel make for each turn of the pedals? Suppose the back wheel has a diameter of 28 inches, what "gear" will this correspond to?

(NOTE.—When a 28-inch wheel makes 2 revolutions for one of the pedals, the gear is said to be 56; *i.e.* 28 × 2.)

2. In a mangle the handle is attached to a wheel of 21-inch diameter; this carries a pinion with 20-teeth gearing with a wheel of 60 teeth cast in one piece with a pinion of 20 teeth; the latter drives another wheel of 60 teeth keyed to the spindle of the roller, and the roller has a diameter of 7 inches. Find the mechanical advantage and how many turns of the handle would be required to pass a tablecloth 4 yards long through the mangle.

3. The handle of a crane works a 1[illegible]-inch crank and carries a pinion with 18-teeth gearing with a wheel of 6 teeth carrying the barrel, whose diameter is 6 inches. Find the mechanical advantage; also how many turns of the handle are necessary to wind in 20 feet of chain.

4. An engine running at 120 revolutions per minute has a driving pulley 24-inch diameter driving a pulley 18 inch diameter on the line shaft. Find the speed of the latter.

5. A machine with a belt pulley 10 inches in diameter runs from the line shaft of the last question. If it is to run at 240 revolutions, what size pulley must be used on the shaft ?

CHAPTER VIII

PULLEYS, BLOCK TACKLE, EFFICIENCY OF MACHINES

A PULLEY consists of a small grooved wheel, called a sheave, pivoted in a kind of frame called the block, to which a hook is attached. A cord or chain is passed through the block so that it lies in the groove about half round the sheave. If the block be suspended from a beam, then a weight attached to one end of the cord can be raised *upwards* by pulling the other end of the cord *downwards* (or in any convenient direction). When used in this way there is no gain in power, because the pull in the two parts of the cord must be the same (if we neglect friction); or if we consider the line PW as a lever, then clearly it has two equal arms. And again, it will be seen that, to raise the weight 1 foot, the power must pull the cord through 1 foot, hence the velocity ratio is 1, and by the principle of equal work the mechanical advantage must also be 1; *i.e.* the power must equal the weight. When used in this way it is called a **fixed pulley**, and it is not used to obtain increase of power but simply to enable one to apply the power in a more convenient direction. Thus a weight may be raised to the top of a building by a man pulling down the rope from below, instead of having to go to the top and pull upwards; or again, a sail may be hoisted by a pulley attached to the mast of a ship.

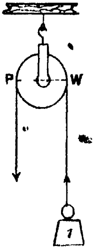

FIG. 32

But the same pulley block may be used in another way, by turning it over and attaching the weight to the block, one end of the rope being fixed to the beam. If we now pull the other end of the rope upwards, the whole block and the weight with it will be raised. Since the block rises with the weight it now forms a **movable pulley**. Now, as before, the pull in the two parts of the rope will be equal, but since both parts share in supporting the weight, the pull in each part will only be *half* the weight, *e.g.* to raise a weight of 2 lbs. a power of only 1 lb. will be required, *i.e.* the mechanical advantage is 2. It must be observed that, in this case, the weight of the block is included and must be added to the load attached to it.

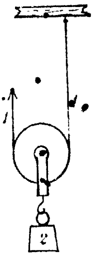

FIG. 33.

Let us now consider the speed ratio. If we wish to raise the weight 1 foot it is clear that *both* parts of the rope must be shortened by 1 foot, and hence there will be 2 feet of rope to pull up; *i.e.* the weight rises only half the distance through which the power moves; hence the speed ratio is $\frac{1}{2}$, and by the principle of equal work the mechanical advantage is again found to be 2. In order to secure the benefit of the downward application of the power, it is usual to combine a movable pulley with a fixed one in one of the ways shown in Fig. 34.

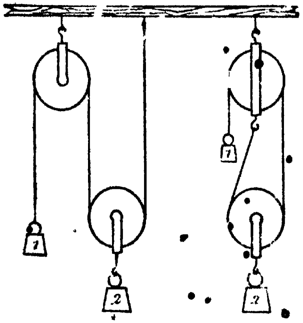

FIG. 34.

A number of movable pulleys may be combined to form a "system" of pulleys. One of the methods of doing this is shown in Fig. 35. It will be seen that there are three movable pulleys, W_1, W_2, and W_3. If we call the weight Q 1 lb., then the first pulley, W_1, divides this equally between two strings, so that the pull on W_2 is only $\frac{1}{2}$: similarly W_2 divides the weight, so that the pull on W_3 is only $\frac{1}{2}$ of $\frac{1}{2}$ or $\frac{1}{4}$, and finally W_3 again divides the weight, so that the pull to be exerted at P will only be $\frac{1}{2}$ of $\frac{1}{4}$ or $\frac{1}{8}$ of the original weight. The weight of the three pulleys and the effect of friction will, of course, increase the power actually required.

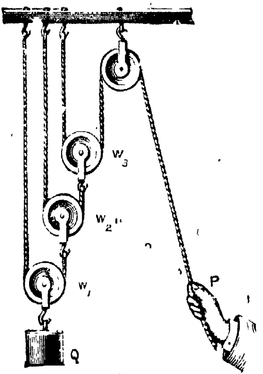

FIG. 35.

The arrangement of pulleys with separate cords is seldom used in practice because the uppermost pulley would reach the top, while the lowest one would only have risen $\frac{1}{4}$ of the distance.

Fig. 36 shows an arrangement with three sheaves in each block and a continuous cord passing round them all.

In this case it will be seen that the lower block and its load are supported by six branches of the cord, and consequently each branch supports one-sixth of the weight, and hence the power to be applied at P will only be $\frac{1}{6}$ of the weight (including the weight of the lower block). The mechanical advantage will thus be 6. The speed ratio, as might be anticipated, will be $\frac{1}{6}$; for if we

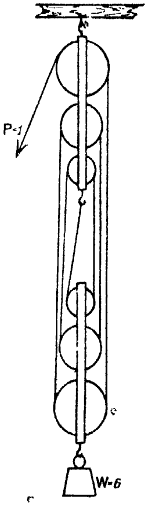

FIG. 36.

vish to raise the weight 1 foot, then each branch of the ord must be shortened by 1 foot, and hence altogether feet of cord must be pulled down.

Instead of placing the sheaves one under another in a ong block, it is more usual to lace them side by side in a vider block, as shown in Fig. 7. This arrangement is more ompact, but the principles involved and the mechanical advantage are just the same as before. Such an arrangement is commonly called a block tackle.

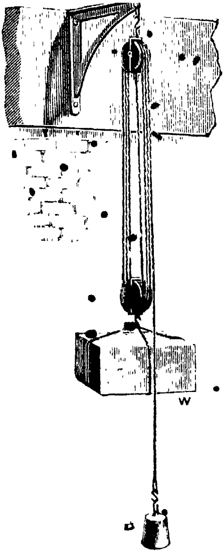

FIG. 37.

Efficiency of Machines.

A machine is simply a contrivance for the convenient application or transmission of power. Thus, the lever, the wheel and axle, and the pulley are machines. In all such machines it will be ound that the power necessary always exceeds, more or less, the amount found by calculation rom the theoretical mechanical advantage; this is due to losses by friction and other resistances. Thus, in a pulley a certain amount of power is consumed in simply turning the sheaves on their axles, and a further amount in overcoming the stiffness of the cord. This power is wasted, and, as a rule, it should be reduced to a minimum by careful design and by lubrication of the moving parts.

It is often instructive to compare the theoretical value of the power required (as calculated from the mechanical advantage) with the actual value as found by experiment.

The relation is generally expressed as a percentage, representing what is called the **efficiency** of the machine.

Thus, if in a certain machine a force of 1 lb. should theoretically raise a load of 10 lbs., whereas it actually raises only 8 lbs., then the efficiency would be $\frac{8}{10}$, or expressed as a percentage, $\frac{8}{10}$ of $100 = 80$ per cent. Or again, if we keep the load of 10 lbs. fixed, and if this would theoretically require a force of 1 lb. but is found to actually need a force of 1·25 lbs., then, since 1·25 lbs. only does the theoretical work of 1 lb., the efficiency is $\frac{1}{1 \cdot 25}$, or, expressed as a percentage, $E = \frac{1}{1 \cdot 25}$ of $100 = 80$ per cent.

Thus, in order to find the efficiency of a machine, all we have to do is, first, to calculate the theoretical power for a certain load, then, secondly, to find by experiment the actual power for the same load.

The efficiency will then be given by the formula:—

$$E = \frac{\text{theoretical power}}{\text{actual power}} \times 100.$$

In finding the actual power care must be taken to make it large enough to actually raise the weight and not merely to balance it, because the friction is only brought into play when the machine moves.

It will often be found that the efficiency varies; thus, in the case of a pulley the rope may be stiffer in one part than another, and the sheaves may move more freely in one position than another; hence it is advisable to make several experiments and then calculate an average value. The state of lubrication will have a marked effect on the result.

In making experiments with a pulley it is convenient to hang a fixed weight on to the block, attach a scale-pan to the power end of the cord, and then place weights in the scale-pan until the weight just begins to rise steadily. (In making calculations the weight of the scale-pan must be added to the weights it contains.)

The Differential Pulley.—This very useful form of pulley consists of an ordinary movable block with a single sheave, A, and a special fixed block, which carries a double sheave made *in one piece* but having two grooves, one of them, C, slightly smaller than the other, B. An *endless* chain passes round the sheaves in the manner shown, so as to leave a free loop, PQ. The grooves are recessed to fit the links of the chain so that the latter cannot move without turning the sheave.

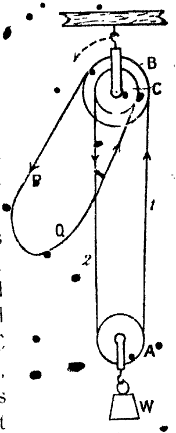

FIG. 38.

Suppose, now, that we pull at P; this will cause B and C to revolve as shown by the arrow. The movement of B will pull up No. 1 branch of the chain and tend to raise W, but the movement of C will *at the same time* lower No. 2 branch, and so tend to lower W. But as B has a larger diameter than C, it follows that No. 1 branch will be raised more than No. 2 is lowered, and the combined effect will be to draw in a length of chain equal to the *difference* between the rise of No. 1 and the fall of No. 2. If 1 foot of chain be drawn in in this way, W will, of course, only rise 6 inches, this amount of shortening taking place in each branch.

Let us now suppose that the *circumference* of the larger groove is 12 inches and that of the smaller groove 11 inches; then if we pull in 12 inches of chain at P, this will cause B and C to revolve just once, and the branches 1 and 2 will be shortened by the difference between 12 and 11 inches, *i.e.* 1 inch, and hence, as above, W will be raised ½ inch. Thus, when P moves 12 inches W moves ½ inch; *i.e.* the speed ratio is $\frac{\frac{1}{2}}{12} = \frac{1}{24}$, and hence the theoretical mechanical advantage would be 24; *i.e.* a force of 1 lb. should balance 24 lbs.

The formula would be:—

$$\text{Mechanical advantage} = 2 \times \frac{\text{circumference of larger groove}}{\text{difference of circum. of grooves}}.$$

One peculiarity of this machine is that the friction is so great that the weight will remain suspended without any power being applied and without the loop PQ being fixed in any way, whereas in an ordinary block tackle the free end of the chain must be held, or fixed to something, if the weight is to be prevented from running down.

On account of this excessive friction the efficiency of this form of pulley is small (in one experiment it was found to be about 30 per cent.); but in many cases this is of small importance compared with the convenience of being able to leave the load hanging in any position without fixing.

In order to lower the load it is necessary to pull the chain at Q.

SUMMARY.

A *fixed* pulley simply changes direction of force, without increasing its effect.

A single *movable* pulley gives mechanical advantage 2.

In ordinary compound pulleys the mechanical advantage number of strings supporting the load.

$$\text{Efficiency of machine} = \frac{\text{theoretical power required}}{\text{actual power required}}.$$

If multiplied by 100 the percentage is obtained.

In differential pulley:—

$$\text{mechanical advantage} = 2 \times \frac{\text{diameter of larger sheave}}{\text{difference of diameters}}.$$

Speed ratio in all cases = mechanical advantage.

EXERCISES.

1. In a common tackle block there are two pulleys in the fixed block and two in the movable block. Find the mechanical advantage.

Suppose it is found that to raise a 56 lb. weight requires a force of 20 lbs., find the percentage efficiency.

2. In a differential pulley it is found that to raise the load 9 inches requires 21 feet of chain to be pulled down. Find the mechanical advantage.

Supposing the efficiency to be 20 per cent., find the force necessary to raise a load of 1 cwt.

3. A movable pulley block weighing 5 lbs. carries three sheaves, and the fixed block also has three sheaves and weighs 6 lbs. Find the theoretical power required to raise a load of 40 lbs., taking account of the weight of the pulley.

4. Sketch a fixed pulley, a movable pulley, and a combination of the two; also a block tackle, with two fixed and two movable sheaves.

PRACTICAL EXERCISES.

1. Fit up a single fixed pulley with a fixed load on one side and a scale-pan on the other, and find what weight (including the weight of scale-pan) is necessary to raise the load *steadily*; hence calculate the percentage efficiency.

Use various loads, and see whether the efficiency varies also.

2. Arrange a single movable pulley with one string fixed to a support and the other end attached to a spring balance reading in grams. Attach a load to the pulley, and examine the reading of the balance, (*a*) when the load is at rest, (*b*) when it is being *steadily* raised, (*c*) when it

is being *steadily* lowered. How do you account for the differences?

3. Arrange a combination of fixed and movable pulleys with a fixed load and a scale-pan, and find the weight required to raise the load steadily; hence calculate the efficiency.

4. Repeat No. 3 with an actual block tackle, using heavier weights, and find the efficiency for various loads.

5. If a differential pulley is available, test its velocity ratio by finding the length of chain to be pulled in to raise the load, say, 1 foot, then find the relation of the load to the power (when the load is being steadily raised), and hence calculate the efficiency.

CHAPTER IX

THE INCLINED PLANE, WEDGE, AND SCREW

In chapter vi. reference has been made to the fact that when a train is going up an incline it is raised more or less quickly according to the steepness of the slope. The steepness or "gradient" is generally expressed by stating the ratio between the vertical rise and the corresponding distance along the incline. Thus a gradient of 1 in 100 means that, in order to rise by a given vertical distance the train must travel 100 times as far along the slope; *e.g.* to rise 1 inch it must travel 100 inches—for a rise of 1 foot or 1 yard, the distances to be travelled will be 100 feet and 100 yards respectively, and so on. A gradient of 1 in 50 would, of course, be steeper, since a given rise is accomplished in half the distance required on the 1 in 100 slope: a gradient of 1 in 50 on an ordinary railway would be considered very steep, whereas 1 in 300 would be a very moderate slope.

It is well known that the gradient of an incline affects the power necessary to mount the slope. Let us consider the relation between the power necessary and the weight to be raised, by applying the *principle of equal work*. Suppose a waggon weighing 1 ton is to be drawn up a slope of 1 in 100. In order to raise the weight through a vertical height of 1 foot the waggon must move along the slope through 100 feet. The work done on the waggon in raising 1 ton through 1 foot will be 1 foot-ton. Hence the work done by the power will also be 1 foot-ton. But since the power acts through a distance of 100 feet, the force will only be $\frac{1}{100}$ of a ton. For the work done by a force of $\frac{1}{100}$ ton acting through 100 feet will be $(\frac{1}{100} \times 100)$ foot-ton = 1 foot-ton.

Thus we get the relation :—

$$\frac{\text{Power}}{\text{Weight}} = \frac{\text{vertical rise}}{\text{length of slope}} = \text{gradient (expressed as a fraction).}$$

Thus with a gradient of 1 in 100 or $\frac{1}{100}$, the power will only require to be $\frac{1}{100}$ of the weight.

The arrangement evidently gives us a mechanical advantage of 100. Thus :—

$$\text{Mechanical advantage} = \frac{\text{weight}}{\text{power}} = 100 = \frac{\text{length of slope}}{\text{vertical rise}}.$$

It must be clearly understood that, in this calculation, we have entirely left out of account any friction, and have only considered the work done in raising the weight against the force of gravity.

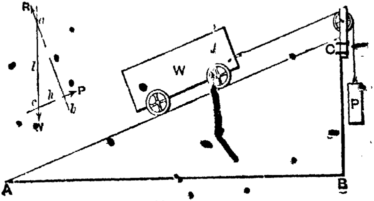

Fig. 39.

Experiments on the relation between power and weight can be readily made with a model inclined plane con sisting of a smooth board AC, which can be tilted at various angles and on which runs a roller or small trolley to represent the "weight," the power being applied by a weight attached to the trolley by a string passing over a pulley at the top of the incline.

It will be found that if the board be tilted so that the vertical rise BC is just half of the length of slope AC, the weight P will support a weight W just twice as great. Thus, if AB = 2 feet and BC = 1 foot, a weight of 1 lb. (P) will support a weight of 2 lbs. (W); but in order that W may be raised 1 foot, clearly P must descend 2 feet. Thus the mechanical advantage is 2.

The student should make a series of experiments, adjusting the inclined plane to various gradients and tabulating the results as follows:—

Weight.	Power.	Mechanical Advantage $\left(\frac{W}{P}\right)$.	Vertical Rise (R).	Length of Slope (L).	Gradient $\left(\frac{R}{L}\right)$.
2 lb.	1 lb.	2	1 foot	2 feet	$\frac{1}{2}$
3 lb.	$\frac{1}{2}$ lb.	6	4 inches ($=\frac{1}{6}$ foot)	2 feet	$\frac{1}{6}$

It will be seen that the mechanical advantage is always represented by a number found by inverting the fraction which represents the gradient.

$$\text{Gradient} = \frac{\text{rise}}{\text{length of slope}}$$

$$\text{Mechanical advantage} = \frac{\text{length of slope}}{\text{rise}}.$$

Example:—

Suppose that a plank 12 feet long be placed against a cart, standing 3 feet above the ground, so as to form an incline, and that a stone weighing 2 cwts. is to be pushed up this incline. What force will be necessary (neglecting friction)?

The gradient is 3 in 12 or 1 in 4; *i.e.* $\frac{1}{4}$.

Hence the mechanical advantage will be 4; *i.e.* the power will be $\frac{1}{4}$ of the weight.

Hence force required will be $\frac{1}{4}$ of 2 cwts = $\frac{1}{2}$ cwt. or 56 lbs.

The Wedge.—If we imagine a heavy stone B resting on a wedge-shaped piece of wood A, it is clear that if A be driven under B by a horizontal blow with a mallet, B would be raised in the direction of the arrow. In this case the weight is not raised by pulling it along the incline, but by driving the wedge (with its inclined faces) under the weight.

This illustrates the relation between the wedge and the inclined plane, but it is more usual to use the wedge for the purpose of producing a splitting action. Thus a downward blow on the wedge C would produce great pressure and so tend to split or rend the object into which it is driven.

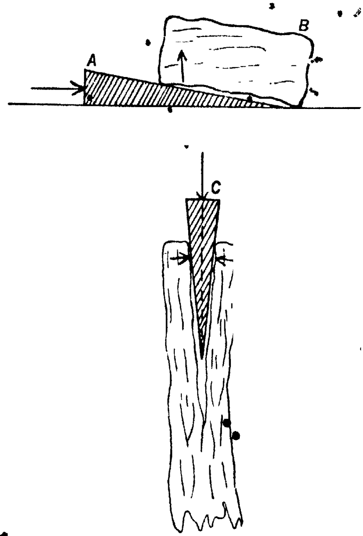

FIG. 10

The height of the ordinary inclined plane here corresponds to the back of the wedge and the length of slope to the length of the wedge.

Thus, mechanical advantage of wedge $= \frac{\text{length of wedge}}{\text{width of back}}$.

Clearly a long narrow wedge will have a greater mechanical advantage than one which is more blunt. Many tools, notably the axe, depend on wedge action. In practice the actual mechanical advantage is much less than the theoretical value on account of the very great friction brought into play, but this friction serves the useful purpose of preventing the wedge from flying out again after the blow.

The Screw.—An ordinary stair may be considered as a stepped incline. To economise space a stair is sometimes made to wind round in a spiral or cork-screw form. The ordinary screw may be considered as an inclined plane wound spirally round a cylinder.

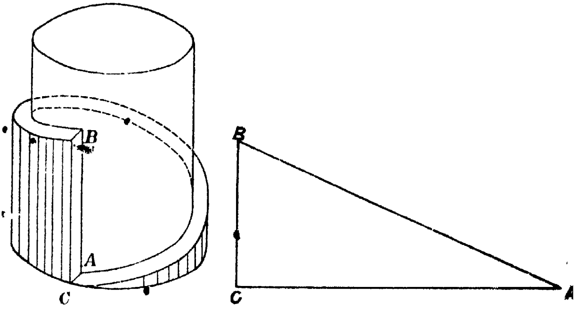

Fig. 41

If we cut a triangle of paper ABC of such a size that the distance AC is just equal to the circumference of a cylinder, the paper may be folded once round the cylinder, and the side AB will then form a winding slope, which, if continued uniformly, would make a spiral round the cylinder.

If we imagine a fly to walk up a pathway following the slope AB, it is clear that, by walking once round the cylinder it would have raised itself vertically through the distance BC; hence it has made use of an incline in which the mechanical advantage is found as usual.

$$\text{Mechanical advantage} = \frac{AB}{BC} = \frac{\text{circumference of cylinder}}{BC}.$$

Now if the spiral be continued, the successive turns will be parallel to each other and BC will represent the *uniform* distance between any two of these successive turns of the spiral. This distance is called the **pitch** of the screw.

$$\text{Hence mechanical advantage of screw} = \frac{\text{circumference}}{\text{pitch}}.$$

The spiral takes the form of a projecting thread, the turns of which are separated by a corresponding groove. The sketch illustrates a screw with a square thread, shown partly in section. In many cases the thread has a V section.

Let us now consider the effect of a screw. If our imaginary fly were to continue its course round and round the spiral pathway the effect would be to carry it up the cylinder in the direction of its axis: thus the nearly circular motion of the fly is converted into a resultant motion along a straight line, forming the axis of the cylinder; and this is precisely the effect of every screw, viz. to convert a circular or turning motion into a motion along the length of the screw. The student should note the following important fact:—

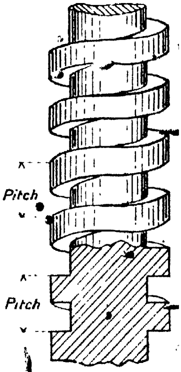

FIG. 42.

One turn of the screw produces linear motion equal to **the pitch of the screw.**

Applications.—In the ordinary applications of the screw it is necessary to have a second hollow screw or "nut" provided with a spiral groove which just fits the thread of the screw itself. Either the screw or the nut may then be caused to travel along by turning one while the other is prevented from turning.

The chief uses are as follows:—

(*a*) To bind or grip things firmly as in the common nut and bolt and in the vice, and the common wood screw. In the latter case the nut is formed automatically by the sharp thread cutting a spiral groove in the wood as it is driven in.

(*b*) To exert great lifting or compressing force as in the screw-jack and screw-press (of which the common letter copying press is an example).

(*c*) To produce a slow and steady motion as in the screw-cutting lathe, where the cutting tool is carried along at a definite rate by the "parent" or "leading" screw while cutting the spiral groove on the screw to be made.

(*d*) In delicate measurements and fine adjustments a *micrometer* screw is frequently used. This consists of a carefully made screw of small pitch, the circular head of which is divided into a number of equal parts, so that a definite fraction of a turn can be made and measured. Thus, for instance, if the pitch be 1 millimetre and the circumference of the head be divided into 100 parts, then each division will correspond to $\frac{1}{100}$ of a turn, and since each whole turn carries the screw forward by a distance equal to the pitch of 1 millimetre, it follows that each division on the head corresponds to a forward motion of the screw of $\frac{1}{100}$ millimetre. The ordinary screw gauge is made on this principle.

In the application of the screw for obtaining great power, it is well to remember that the actual mechanical advantage is much smaller than the theoretical value: in other words, the efficiency is small, in consequence of the great friction involved. In many cases this friction is a great advantage, since it counteracts the tendency of the screw to loosen itself by turning back when the power ceases to be applied. It is this friction which prevents a nut and bolt from becoming unscrewed, but it is well known that vibration may cause unscrewing to take place slowly in spite of this friction, and in some cases it is necessary to use a lock-nut, or some other device, to prevent it.

In using a screw it is practically always turned by some form of lever handle, as in the ordinary spanner used in tightening a nut and bolt, and in the handle of a vice or screw press. In such cases the length of this handle must be taken into account in estimating the mechanical advantage. The simplest plan is to substitute the circumference of the circle described by the end of the handle for the circumference of the cylinder in the formula previously given, since this circle gives the distance through which the power moves.

The formula then becomes:

$$\left.\begin{array}{l}\text{Mechanical advantage of}\\ \text{screw and lever handle}\end{array}\right\} = \frac{\text{circumference of circle described by handle}}{\text{pitch of screw}}.$$

Example :—

In a screw-jack the pitch of the screw $= \frac{1}{2}$ inch, the length of the handle is 14 inches. Find the power necessary to raise 1 ton.

Circumference of circle described by handle $= 28 \times 3\frac{1}{7} = 88$ inches.

Hence mechanical advantage $= 88 \div \frac{1}{2} = 88 \times \frac{2}{1} = 176$.

Therefore power required $= \frac{1}{176}$ of the weight $= \frac{1}{176}$ of 2240 lbs. $= 12\frac{8}{11}$ lbs.

This is purely a theoretical result, for in the case of a screw, owing to the great pressure between the threads of the screw and the nut, there is great friction, and hence the efficiency is small; *i.e.* a power considerably larger than the theoretical value will be necessary.

It will be a useful exercise for the student to determine the efficiency of a screw by actual experiment, as in the case of the pulley.

A small screw-jack carrying a heavy weight (say, 56 lbs.) may be used, the power being applied through a spring balance attached to the handle, the pull being continued until the screw moves steadily. Care must be taken to keep the direction of pull at right angles to the handle.

SUMMARY.

In the inclined plane :—

$$\text{Mechanical advantage} = \frac{\text{length of slope}}{\text{vertical rise}}.$$

A gradient of 1 in 100 means that the length of slope is 100 times the rise.

$$\text{Mechanical advantage of wedge} = \frac{\text{length of wedge}}{\text{width of back}}.$$

$$\text{Mechanical advantage of screw} = \frac{\text{circumference}}{\text{pitch}}.$$

One turn of screw gives travel equal to pitch.

$$\left.\begin{array}{l}\text{Mechanical advantage of combination}\\ \text{of screw and lever handle}\end{array}\right\} = \frac{2\pi \times \text{length of handle}}{\text{pitch}}$$

EXERCISES.

1. Neglecting friction, what pull must an engine exert on a train weighing 200 tons on a slope of 1 in 150?

2. With a gradient of 1 in 80, how many feet will a railway line rise in a mile?

3. A micrometer screw has a pitch of $\frac{1}{25}$ inch. How far will it travel along its axis when turned through 1·81 revolutions?

4. A screw-jack is worked with a handle 3 feet long and the pitch of the screw is $\frac{1}{4}$ inch. What total pressure will be exerted when a pull of 40 lbs. is applied to the handle?

5. A bolt having 10 threads to the inch is turned with a spanner 8 inches long. What is the mechanical advantage?

PRACTICAL EXERCISES.

1. If a model inclined plane is available, find the relation between power and load and between length and vertical rise of the plane for various gradients.

In order to partly neutralise the effect of friction, find first the weight which will just raise the load steadily, then the weight which will just let it run down steadily, and take the mean.

2. Measure the pitch of the screw in a screw-jack and the length of its handle; support a heavy weight (say, 50 lbs.) upon it, and by means of a cord attached to the end of the handle, passing over a pulley and carrying a scale-pan, find what weight is required to turn the screw steadily. Calculate the theoretical and actual mechanical advantage and find the efficiency.

(NOTE.—In order to keep the cord pulling in the right direction it may be attached to a grooved wooden disc fastened to the handle.)

3. Examine a micrometer screw gauge, note the pitch of the screw and the number of divisions on the head; hence find the value of each division, and use the instrument to find the thickness of various wires and plates, of metal or glass.

CHAPTER X

PARALLEL FORCES—CENTRE OF GRAVITY

SUPPOSE we have a beam 3 yards long resting upon supports at A and B, and carrying a load of 3 cwts. at a distance of 1 yard from A; and for the present let us consider the beam as having no weight. The downward pull of the weight is balanced by the upward pressures or reactions, as they are called, of the supports at A and B. It should be clear that as the three forces are all acting vertically they must be parallel to each other, and the sum of the two upward forces must be just equal to the downward force.

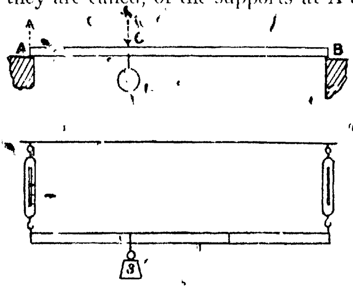

FIG. 43.

If the load were placed half-way between A and B the weight would clearly be divided equally between the two supports; but as it is arranged nearer to A it seems probable that A will have to bear the greater part of the load; let us now consider how it will be distributed.

The beam may be compared with a lever, each support in turn being considered as the fixed fulcrum. Thus, if A be considered as the fulcrum, C will be the weight, and the power will be represented by the upward pressure at B. The tendency of the force at C will be to turn the beam clockwise, *i.e.*, to force the end B downward. The moment of this force about A will be $3 \times 1 = 3$ cwt. yard units.

This tendency must be resisted by an equal anti-clockwise moment, due to the upward pressure at B.

Let this upward force be P_1. Then the moment of this force about A will be $P_1 \times 3 = 3\,P_1$ cwt. yard units. But as the moments are equal we have

$$3\,P_1 = 3.$$

Hence $P_1 = 1$ cwt.

Now let us regard B as the fixed fulcrum and the reaction at A as the force opposing the weight at C. The moment of the force at C about B will be

$3 \times 2 = 6$ cwt. yard units.

If the reaction at $A = P_2$ its moment about $B = P_2 \times 3 = 3 P_2$ cwt. yard units. Hence $3 P_2 = 6$ and $P_2 = 2$ cwts.

Thus the reactions at A and B are 2 cwts. and 1 cwt. respectively, and the sum of these forces is equal to the weight C, viz. 3 cwts. Again, it will be noticed that B is *twice* as far away from C as A is, and that it bears *half* as much of the weight as A. In this case it appears that the weights borne by A and B are *inversely* proportional to their distances from C, and experiment will show that this is a general rule, when a single load is distributed between two supports.

Experiments of this kind are readily made by suspending a wooden rod with a spring balance at each end (as shown in the figure), taking care that both balances hang vertically, so that they pull in parallel directions; a weight to represent the load, can be hung by string from any part of the rod. If the readings of the balances be taken before attaching the weight, allowance can be made for the weight of the rod by deduction from the final readings. The student should make a series of experiments with the weight in various positions along the beam.

The results may be recorded as follows :—

(R*a* denotes the reaction at A, and R*b* the reaction at B.)

Taking Moments about A.						Taking Moments about B.					
Weight.	Distance from A	Moment.	Distance BA.	R*b* by Calculation.	R*b* by Balance.	Weight.	Distance from B.	Moment.	Distance BA	R*a* by Calculation.	R*a* by Balance.
3	1	3	3	1	?	3	2	6	3	2	?

Let us now consider the weight of the beam itself, or the wooden rod which represents it in the experiment.

If the beam be of uniform thickness and density its weight will clearly be equally divided between the two supports, and each will bear one half of its weight. In the example given at the beginning of the chapter, if we imagine the weight of the beam itself to be 1 cwt. we shall simply have to add ½ cwt. to the amounts already calculated as the reactions at A and B respectively. The effect of the beam is in fact exactly the same as if its whole weight were concentrated at the middle.

If we have several loads in different positions on the same beam we can very easily find the total effect by taking the moment of each load separately, about one end of the beam, and adding the results together; the total will be equal to the moment of the force of reaction; and, knowing the distance at which it acts, we can find the value of the force.

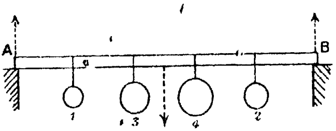

Fig. 44

Example :—

Suppose a beam 5 yards long and weighing 1 cwt. to carry weights of 1, 3, 4, and 2 cwts. at equal distances of 1 yard as shown in the figure.

To find the reaction at B, take moments about A.

Weight No. 1 [illegible] cwt. at 1 yd. Hence moment = 1 cwt. yd. units
" No. 2 = 3 cwt. at 2 yds. Hence moment = 6 " "
" No. 3 = [illegible] cwt. at 3 yds. Hence moment = 12 " "
" No. 4 = 2 cwt. at 4 yds. Hence moment = 8 " "
Beam = 1 cwt. at 2½ yds. Hence moment = 2½ " "

Therefore total clockwise moment about A = 29½ cwt. yds.

Thus the anti-clockwise moment produced by the reaction at B must also be 29½ cwt. yards.

Let the reaction be R, its distance = 5 yds. Hence moment = 5 × R.

$$\text{Therefore } 5 \times R = 29\tfrac{1}{2}$$

$$\text{and } R = \frac{29\frac{1}{2}}{5} = 5{\cdot}9 \text{ cwts.}$$

The reaction at the end A is found by remembering that the sum of the reactions at A and B must equal the total weight.

$$\text{The total weight} = 1 + 3 + 4 + 2 + \tfrac{1}{2} = 10{\cdot}5 \text{ cwt.}$$
$$\text{Reaction at B} = 5{\cdot}9 \text{ cwt.}$$

$$\text{Hence reaction at A} = \underline{4{\cdot}6} \text{ cwt.}$$

The student should verify this method of working by experiments with the wooden rod and spring balances, using several weights.

CENTRE OF PARALLEL FORCES.

So far we have been considering the distribution of weight between *two* points of support; but it is also possible to find a position in which a single support will maintain a body without its having any tendency to turn round. Thus if we imagine a weightless rod AB with weights of 1 lb. and 2 lbs. at the ends, the total downward force will be 3 lbs. This can be balanced by an upward force of 3 lbs.; but if we apply this upward force at the middle of the rod (say, by hanging it by the middle from a

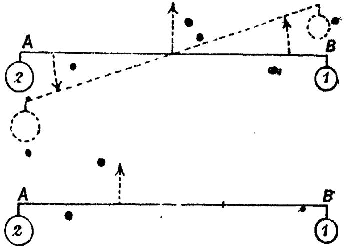

FIG. 45.

spring balance), the greater force at A will cause the rod to swing round as indicated by the curved arrows, because the moment about C of the force at A will exceed the moment of the force at B. But if we remove the point of support to a point $\frac{1}{3}$ of the length of the rod from A and $\frac{2}{3}$ of its length from B, then the moment about C of the forces at A and B will be equal, and, since they tend to turn the rod in *opposite* directions, they will neutralise each other, and the rod will have no tendency to swing round. Thus a single upward force of 3 lbs. at C will balance or be in equilibrium with the two downward forces of 2 lbs. at A and 1 lb. at B. It is clear that a single downward force of 3 lbs. at C would also be in equilibrium with an upward force of 3 lbs., and thus the single 3 lb. downward force at C has the same *effect* as the two forces at A and B. A single force which, in this way, has the same effect as two or more separate forces acting together is called the **resultant** of those forces. Thus the *downward* force of 3 lbs. acting at C is the resultant of a 2-lb. force at A plus a 1-lb. force at B. The *upward* force at C which balances the forces at A and B is thus equal and opposite to the resultant and acts from the same point. It should be noted that the two forces A and B are parallel forces, and the point C from which the resultant of these forces acts is sometimes spoken of as the "centre" of the parallel forces.

A centre of action and resultant could be found for any number of parallel forces.

The student may try the experiment of suspending a wooden rod from a spring balance by means of a kind of stirrup of wire, and after hanging two or more weights from the rod, finding by trial the position of the stirrup about which the loaded rod will balance. He should then find the moments on both sides of the point of support and convince himself that the two sets of moments are equal.

CENTRE OF GRAVITY.

Any body may be imagined to be made up of innumerable particles, each attracted by the earth, and thus its weight may be considered as made up of a series of parallel forces represented by the weights of the individual particles. It should be possible to find a resultant and centre of action of all these parallel forces, and to balance this resultant by an equal and opposite force acting at the same point. If we suspend the body from such a point it will have no tendency to swing round in whatever position it is placed. This point is called the **Centre of gravity**; which may therefore be defined as the centre of action of the parallel forces representing the weights of different parts of the body, or more simply as that point from which, if the body be suspended, it will remain in equilibrium in any position without any tendency to swing round or topple over.

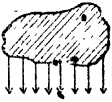

FIG. 46.

To Find the Centre of Gravity.—In the case of a regular symmetrical solid it is generally easy to see where the centre of gravity will be, for a point can be found such that every particle on one side of that point is balanced by a corresponding and equidistant particle on the other side. Thus the centre of gravity of a sphere or ball is evidently at its centre. The centre of gravity of a flat circular body or disc is at the centre of the circle; that of a square or rectangle at the point where the diagonals intersect, and so on. The centre of gravity of a ring would be at the centre of the ring, and this illustrates the fact that the centre of gravity need not be in the actual body itself.

In the case of a flat irregular body (say, a piece of card or tinplate) the centre of gravity may be found experimentally by suspending the body from various points and noting the position in which it hangs freely. Thus, if it be suspended by any point A it will swing round until the centre of gravity, C, comes vertically under A, and if by means of a plumb-line we mark the vertical dotted line, the centre of gravity must be somewhere in this line. If we now suspend it from another point B, and again mark a vertical dotted line, the centre of gravity will be in this line also, and hence it can only be at the point where the two lines cross. The body may be suspended from a third point, and the third vertical line should cross the other two at the same point.

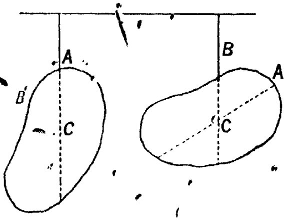

FIG. 47.

Having found the centre of gravity in this way, the accuracy of the determination may be tested by drilling a small hole at the point and suspending the body by a thread passed through the hole and knotted on the other side. The body should hang horizontally without any tendency to overbalance or slope down to one side.

Stability.—If a cone stands upon its base it is not easily knocked over, because if slightly displaced it tends to fall back on to the base. It is said to be in *stable* equilibrium. If the cone be laid on its side, a slight force will cause it to roll, and it will not tend to fall back, but will remain indifferently in any position. It is said to be in *neutral* equilibrium. If we were skilful enough to balance a cone upon its point, the slightest displacement would cause it to fall over. When balanced thus it is said to be in *unstable* equilibrium.

The question of stability depends on the position of the centre of gravity with regard to the base on which the object stands. If a slight displacement causes the centre of gravity to rise, then the body will tend to fall back to its original position; but if it be displaced so far that a vertical line from the centre of gravity falls outside the limits of the supporting base, then it will fall over into a new position.

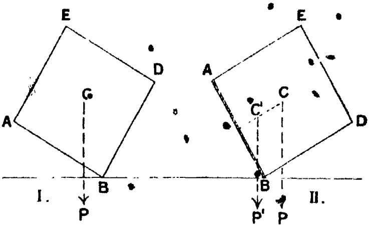

FIG. 48.

Thus, if ABDE represents one face of a cube, a slight displacement as in I. will not cause it to fall over; it will rather tend to fall back on to the side AB. But a greater displacement, as in II., which brings the vertical line CP beyond B will cause it to fall over on to another side BD.

If the centre of gravity be high up, the body becomes top-heavy and will more readily fall over. Thus a cart carrying a big load of hay, or an omnibus crowded with people on the top, will be more easily overturned on an uneven road than an empty vehicle.

To secure stability in a body the weight should be arranged as low down as possible.

Thus, if the face AB in the above cube be weighted with sheet lead so as to bring the centre of gravity down to C′, then since the line C′P′ does not fall beyond B the cube will not overbalance in the position shown, but will tend to fall back on to the face AB.

SUMMARY.

1. Distribution of weight of loaded beam is found by taking moments about each point of support in turn.

2. With a single load it is divided between the two supports *inversely* as their distances

3. The resultant of a number of parallel forces is equal to their sum (or difference if in opposite directions), and acts from a fixed point called the "centre" of the parallel forces.

4. The centre of gravity of a body is the centre of action of the parallel forces, due to the weights of the particles of the body.

5. When a body is freely suspended it hangs so that the centre of gravity falls vertically below the point of support.

6. If hung from the centre of gravity, it hangs indifferently in any position.

7. The stability of a body depends on the position of the centre of gravity in relation to its base of support.

8. The lower the centre of gravity the greater the stability.

EXERCISES.

1. A beam 15 feet long, and weighing $\frac{1}{2}$ cwt., carries a load of 2 cwts. 5 feet from one end. Find the reactions on the supports situated at the ends of the beam.

2. If the above beam carries an additional load of 1 cwt. 5 feet from the other end, what will the reactions then be?

3. A weightless rod 5 feet long carries a weight of $1\frac{1}{2}$ lb. at the end A, and one of $3\frac{1}{2}$ lbs. at the end B. Find the distance of the centre of gravity from A.

4. Show by diagrams the position of the centre of gravity of a square, a rectangle, a circle, a ring, a cylinder.

PRACTICAL EXERCISES.

1. By means of a wooden rod and spring balances make experiments on parallel forces as described in the chapter.

2. Attach unequal weights to the ends of a wooden rod, and find the point of support about which it will balance. Do your results agree with the law of moments?

3. Determine the centres of gravity of various regular and irregular pieces of cardboard or sheet metal by suspending from various points, and using a plumb-line to draw vertical lines from the points of support.

4. Determine the limiting angle of stability of a wooden block, (*a*) alone, (*b*) when weighted at the bottom with sheet lead, (*c*) when weighted at the top with sheet lead.

CHAPTER XI

PARALLELOGRAM AND TRIANGLE OF FORCES.

THE figure represents a river 4 furlongs (*i.e.* ½ mile) wide. Let us suppose that a boatman starts from A, and rows as if going straight across to B at such a rate that, if there were no current, he would reach B in 10 minutes. Now, let us imagine that the current is flowing in the direction of the arrow at the rate of 3 furlongs in 10 minutes, so that if the boat had been allowed to drift it would in the 10 minutes have reached the point C, 3 furlongs down stream; it is reasonable to suppose that

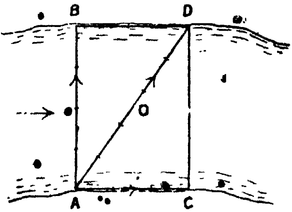

FIG. 49.

the *combined* action of the rowing and the stream would be to bring the boat in 10 minutes to a point D, opposite to C. At the end of 5 minutes the boat would be half-way across and also half-way down between the lines AB and CD, viz. at the point O. In fact, it is not difficult to see that at any time the boat would be somewhere on the line AD, which forms the diagonal of the figure AD, which line, therefore, represents the actual course of the boat.[1]

Thus, it appears that the combined effect of the two motions, viz. the motion in the direction AB produced by the rowing, and that in the direction AC caused by the stream, has been a motion in the *direction* of the diagonal of the figure. Further, the *length* of this diagonal (which happens to be 5 furlongs) will represent the actual distance covered by the boat in 10 minutes, hence its actual velocity has been 5 furlongs per 10 minutes. Thus, the diagonal indicates both the **velocity** and the **direction** of the motion of the boat, this motion being the combined effect or **resultant** of the two motions represented by AB and AC.

Again, we may observe that the boat has been subject to two *forces*, viz. the force due to the rowing, urging it in the *direction* AB, and the force due to the stream, urging it in the *direction* AC, the *resultant* of these two forces has been a force driving it in the *direction* AD.

Further, if we consider the force of the rowing as being equal to 4 units (since it produced a velocity of 4 furlongs per 10 minutes), we may regard the force of the stream as being equal to 3 units (since it would produce a velocity 3), and the resultant force as 5 units (since it causes a resultant velocity 5). Now, if the line AB be 1 inch

---

[1] This combination of two motions can be illustrated experimentally by placing a marble in a glass tube 2 or 3 feet long, and by giving the tube a sideways motion across a table, and, at the same time, inclining the tube to make the marble roll along it, the marble may be made to trace a diagonal line

(*i.e.* 4 quarter inches) long, we may regard the 4 units of length as representing the 4 units of force.

Similarly, AC being 3 quarters of an inch long would represent the 3 units of force in the direction AC and AD, 5 quarter inches long would indicate a force of 5 units in the direction AD.

Thus we may regard the lines AB and AC (forming two sides of the parallelogram) as representing the two forces acting on the boat both in *direction* and in *magnitude* (*i.e.* strength), and the line AD (forming the diagonal of the parallelogram) as similarly representing the resultant force both in direction and magnitude.

Let us now see whether a similar construction will give us a graphic representation of two forces, whose resultant is balanced or held in equilibrium by a third force, so that no motion is produced.

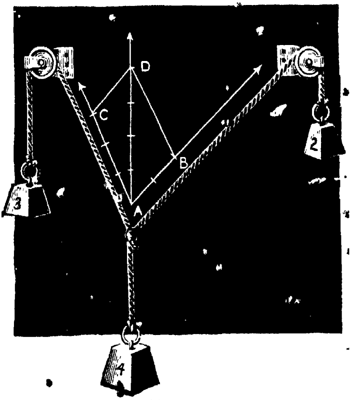

FIG. 50.

The figure represents two weights of 2 lbs. and 3 lbs. attached to cords passing over pulleys so that they pull in the directions AB and AC on the knot A. These two forces are balanced by a third force of 4 lbs. acting vertically *downwards*. But since 4 lbs. acting downwards would just balance a force of 4 lbs. acting upwards, it follows that the combined effect of the forces of 2 lbs. and 3 lbs. acting obliquely must be just the same as that of a single force of 4 lbs. acting vertically upwards, hence this

4 lbs. upward force is the resultant of the two oblique forces. Now in the figure a parallelogram ABCD has been constructed in such a way that the sides AB (2 units long) and AC (3 units long) represent the oblique forces in magnitude and direction, and it will be seen that the diagonal, which is 4 units long and which is vertical, just represents the upward resultant both in magnitude and direction. It should be pointed out, and proved by actual trial, that if we displace the knot A it will always return to the same position, if the pulleys move freely enough, because only in this position will the direction of the oblique forces be such as to give a resultant in equilibrium with the downward pull of the 4 lbs. weight.

We can now formulate the following important law:—

The Parallelogram of Forces.

> If two forces, acting at the same point on a body, be represented, in magnitude and direction, by two neighbouring sides of a parallelogram, then the resultant of these forces will be represented, in magnitude and direction, by that diagonal of the parallelogram, which passes through the point at which the forces are applied.

This law should be carefully distinguished from the law of parallel forces, referred to in the last chapter. The resultant of any number of parallel forces is equal to their sum (unless acting in opposite directions, when it is the difference). In the parallelogram law we are dealing with forces which are *not* parallel, and the resultant is *not* equal to the sum of the two forces; thus in the last example the forces of 2 and 3 lbs. have a resultant equal to 4 (not 5) lbs.

The parallelogram law can be applied in two ways.

(*a*) We may suppose two forces to be combined and replaced by their resultant. This is called the **composition** of forces.

(b) We may regard a single force as being the resultant of two forces acting in certain directions, and we may suppose the single force to be replaced by these two component forces. This is called the **resolution** of forces.

A good example of resolution of forces is afforded by the inclined plane. Thus, suppose a weight a, of 1 lb., to rest on a plane, whose gradient is 1 in 2, so that AC is twice BC.

Let the vertical dotted line ac represent in direction the force due to the weight a, and let it be, say, 1 inch long to represent the weight of 1 lb.

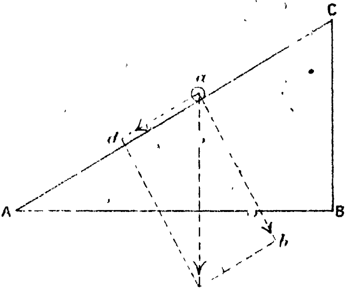

FIG. 51.

We may imagine this single force to be made up of two components, one acting in the direction ad, and tending to carry the weight directly down the incline; and the other acting in the direction ab, at *right angles* to the face of the incline, which latter force is directly balanced by the reaction or resistance of the plane itself. Now, having decided on the directions of ad and ab, let us arrange their lengths so that they form a parallelogram of which ac is the diagonal. This is easily done by drawing lines cd and cb parallel to ad and ab until the lines meet.

As ac has been made 1 unit long to represent the weight of 1 lb., the lengths of ad and ab will indicate the magnitudes of the component forces. But clearly cb is just as long as ad, thus we can take the sides of the triangle abc as representing the relative magnitudes of the three forces. It can easily be shown by geometry that this small triangle *abc* is exactly similar in shape to the triangle ABC (although turned into a different position), and hence the relation of *cb* to *ac* will be just the same as that of CB to AC, *i.e.* *cb* will be just half of *ac*. This means that the power required to draw the load up the incline is ½ the weight, *i.e.* the mechanical advantage is 2; which is precisely the same result as we obtained in chapter ix. by the application of the principle of equal work.

The Triangle of Forces.—The student will have observed that in the last example we made use of the triangle *abc* instead of the whole parallelogram. It is easy, in fact, to obtain a very useful modification of the parallelogram law. Thus if AB and AC represent two forces and AD their resultant, then if we reverse AD turning it into AE, we shall have three forces in equilibrium; and since BD is equal and parallel to AC it follows that the three sides of the triangle ABD will be parallel to the directions of the three forces in equilibrium, and their lengths will be proportional to the magnitudes of the forces.

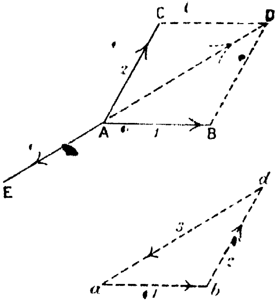

Fig. 52.

We must be careful to note in which direction along each line the force acts. In the separate triangle *abc* the sides are numbered to correspond with the forces and the directions are indicated by arrows. It will be seen that these directions are followed by going round the sides of the triangle *in order and in the same direction.*

Thus we may state the following law:—

Any three forces acting at a point, which are in equilibrium, may be represented in magnitude and direction by the three sides of a triangle taken in order, those sides being parallel to the directions of the three forces.

The law may be easily verified by attaching three spring balances by means of strings to a central knot and stretching them to three nails arranged on a board. The balances indicate the magnitudes of the three forces, and the directions of the strings may be traced on to a piece of paper.

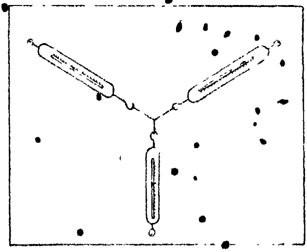

FIG. 53.

It will be observed that although the sides of the triangle are parallel to the three forces, they do not, as in the parallelogram law, pass through the same point as the forces themselves do. For this reason it is very often convenient to draw two diagrams, one a structure diagram, representing the disposition of the forces, and the other the triangle whose sides are parallel to the forces.

This law has most important applications in calculating the forces which come into play in such structures as bridges, roof-trusses, and built-up structures generally.

Let us examine two simple applications.

The Jib Crane.—Suppose PQ represents the jib and PR the jib-tie of a crane which is supporting a load W. A little reflection will show that the effect of W will be to exert a pull in PR and a push or thrust in PQ, and consequently the jib will react with a push towards P and the tie will react with a pull towards R, as indicated by the arrows, and the three forces will be in equilibrium. If we construct the triangle ABC with sides parallel to the three forces and make AB of such length that it represents the weight, then the length of AC will indicate the pull or tension on the tie, and the length of BC will give the push or thrust on the jib. The accuracy of the results may be tested experimentally by means of a model crane, the pull in the tie being measured by an ordinary spring balance, and the thrust in the jib by a special form of balance for measuring a push.

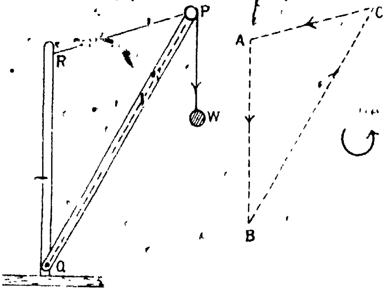

Fig. 54.

A Simple Roof-truss.—Let ABC represent a model truss, AB and AC being the principal rafters and BC a tie-rod.

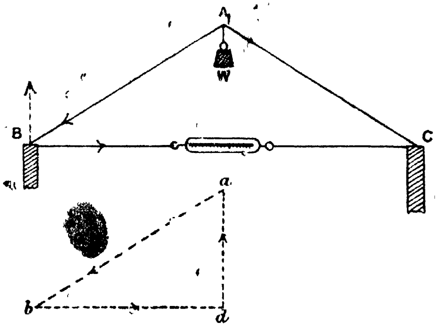

Fig. 55

Suppose a weight of 1 cwt. to be hung at A. This weight will be equally divided between the two walls at B and C.

Hence the reaction or upward force at B will be 56 lbs. The other forces at B will be the thrust of the beam AB and the pull of the tie-rod BC, and these three forces are in equilibrium, hence we may construct the triangle of forces *abd*, making *ad* of such length as to represent the ½ cwt. upward force at B. Then the length of *bd* will indicate the pull on the tie, and the length of *ab* the thrust on the beam. By introducing a spring balance in the tie on the model the above result may be verified by experiment.

SUMMARY.

1. *Parallelogram Law.*—If two sides of a parallelogram represent two forces, the diagonal represents the resultant in magnitude and direction.

2. Resolution of forces = Replacing the resultant by its components.

Composition of forces = Replacing two forces by the resultant.

3. *Triangle of Forces.*—Three forces in *equilibrium* represented by three sides of triangle in *order*.

EXERCISES.

1. Find (*a*) by drawing, and (*b*) by calculation, the resultant of two forces of 12 lbs. and 16 lbs. acting at right angles to each other.

2. Draw a force diagram for a jib crane, the jib being inclined at an angle of 60° with the ground and the jib-tie making an angle of 30° with the jib. Supposing a weight of 50 lbs. hung from the jib, find the pull on the tie and the compression of the jib.

3. Draw a diagram of a simple roof-truss, the rafters being inclined at an angle of 30° with the horizontal. Supposing a weight of 2 cwts. suspended from the ridge, find the reaction at each wall of support and the pull in the tie-rod.

4. Draw a diagram to illustrate the actual direction taken by a boat urged *straight* across stream at 4 miles per hour, supposing a current of 2 miles per hour to act on it. In what direction should the boat be urged in order to travel straight across stream?

PRACTICAL EXERCISES.

1. Test the truth of the parallelogram law, by using two weights slung over good brass or aluminium pulleys with a third weight hanging from the cord between the pulleys. Vary the size of the third weight, and note the effect on the shape of the parallelogram.

2. Verify the same law by using a heavy weight slung from two spring balances suspended from separate hooks. Unless a fairly heavy weight is used, the weight of the spring balances interferes with the accuracy of the result.

3. Attach three spring balances to three strings tied to a common ring. Stretch the balances to three nails in a horizontal board, note the pull on each and the angles between the strings, and hence verify the law of the triangle of forces.

CHAPTER XII

FLUID PRESSURE

Liquids Transmit Pressure in all Directions.—In chapter i. it has been pointed out that the particles of a liquid are free to move in all directions. It follows from this that if pressure be exerted, in any direction, upon a confined mass of liquid, it may relieve itself by moving in any other direction. Thus, if we have a vessel full of water, and with a number of movable plugs or pistons, A, B, C, D; then, if we exert a downward pressure on the piston A, the liquid will try to escape by pushing the piston

B downwards, but it will also push the piston C sideways, and the piston D upwards, and, further, it will exert pressure in all these directions at one and the same time.

This may be illustrated by taking a vessel full of water, and provided with a number of small holes in different places. If we exert pressure on the water by means of a piston, it will be found that the water will escape forcibly at all the holes, showing that the pressure of the liquid causes it to push in all directions against the walls of the vessel.

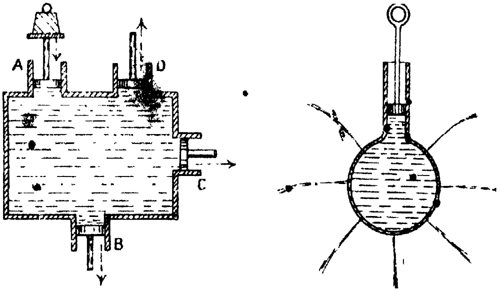

FIG. 56.

Pressure Proportional to Area.—We have seen that a liquid can exert pressure upon a number of pistons at the same time. If these pistons be equal in area, and if we connect two of them together, the pressure on the two will be double the pressure on the single piston. If the two pistons be replaced by a single one, of double the area, the pressure will still be double, and, similarly, it follows that if we further enlarge the area of the piston, the pressure upon it will increase in the same proportion.

This fact could be proved by having a small piston, *b*, and a larger one, *a*, attached to the same vessel, *A*, full of water; a small weight, *d*, acting upon *b* would be found to balance a larger weight, *c*, pressing upon *a*, and, if the areas of *a* and *b* were measured, it would be found that the weight *c* was as many times greater than *d* as the area of *a* is greater than the area of *b*. The above arrangement is clearly capable of giving us increased power, or, in other words, it forms a machine with a mechanical advantage. Thus, if the diameter of *a* is twice that of *b*, then the area of *a* will be four times that of *b*, and a weight of 1 lb. placed at *d* will sustain a weight of 4 lbs. at *c*, *i.e.* the mechanical advantage is 4. If *d* pushes *b* downwards, then *a* will be forced up, and will raise *c*. Suppose *b* to fall 1 foot, so that *d* does 1 foot-pound of work; then *a* will only be raised $\frac{1}{4}$ foot, for the water pushed out of *b*'s cylinder will only rise one-quarter as far in *a*'s cylinder, since the latter has four times the area of cross section. Thus, the 4-lb. weight *c* will only rise $\frac{1}{4}$ foot, the work done on it will therefore be $4 \times \frac{1}{4} = 1$ foot-pound, so that here also, as in the case of previous machines, the principle of equal work applies.

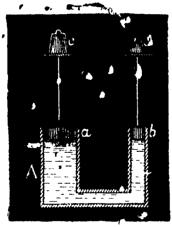

FIG. 57.

Since the pressure of a liquid is proportional to the area on which it acts, such pressures are stated at so much per unit of area; *e.g.* we may speak of a pressure of 10 lbs. per square inch, or one of 5 tons per square foot, or one of 100 grams per square centimetre. These are, of course, not equivalent to each other.

The Hydraulic Press.—The above method of multiplying pressure is applied in many hydraulic machines in which great pressure is required; *e.g.* the Bramah or Hydraulic Press, the Hydraulic Lifting Jack, the Hydraulic Punching Bear, &c. The diagram shows a hydraulic press, partly in section. By means of a lever handle pressure is exerted on the plunger S, which has only a small diameter. This pressure is transmitted to the plunger or ram t, having a much larger diameter, so that the pressure is multiplied many times. A single stroke of S would only raise t by a very short distance, hence valves[1] are provided, and, when S is raised, it draws in more water from the tank A, without allowing any water to return from X; in this way t may be raised as far as necessary by repeated strokes of S.

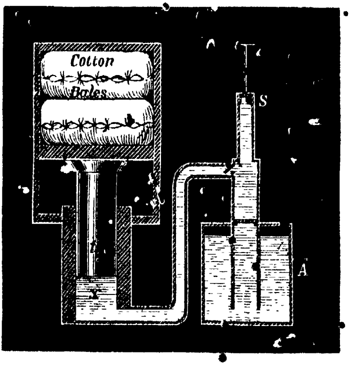

FIG. 58

To find the total mechanical advantage, we must take account both of the leverage of the handle, and also of the areas of the two pistons.

Example :—

Suppose the area of the small plunger to be 1 square inch, and that of the ram 1 square foot, and that the lever handle gives a mechanical advantage 10.

$$\left.\begin{array}{r}\text{Mechanical advantage}\\ \text{due to water pressure}\end{array}\right\} = \frac{1\text{ sq. foot}}{1\text{ sq. inch}} = \frac{144}{1} = 144.$$

$$\text{Mechanical advantage due to handle} = 10.$$

$$\text{Total mechanical advantage} = 144 \times 10 = 1440.$$

Thus, a pressure of 1 lb. on the handle would produce a pressure on the ram of 1440 lbs. (very nearly 13 cwts.).

---

[1] The mode of action of pump-valves is described in chapter xiv.

Such presses are much used for compressing cotton and other goods into bales for shipment.

Hydraulic power is also used in large engineering works for operations where powerful, steady pressure is required; *e.g.* in punching, riveting, bending, and cutting iron plates; also in forming cold-drawn lead pipes, &c.

Pressure Caused by Weight of Liquid.—It is clear that a liquid may exert pressure on account of its own weight. Thus, for instance, if we have a cylinder whose open bottom is closed water-tight by a light plate (*a*) held in position by a string, and if we place 1 lb. of water in this, the string would have to be pulled up with a force of 1 lb. (plus the weight of *a*), in order to resist the downward pressure of the water. If we now sink the cylinder in a larger vessel of water, until the level outside is equal to the inside level, then we may release the string without the water having any tendency to run out of the cylinder; hence the downward pressure on the plate must be balanced by an equal upthrust of the water in the vessel. It should be observed that the amount of this upthrust does not depend on the size or shape of the outer vessel; thus, although it may contain much more than a pound of water above the level of the plate, its pressure on the latter will not exceed one pound, for, if we attempt to pour any more water into the cylinder, the plate will be at once pushed away.

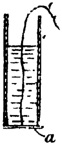

FIG. 59.

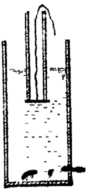

FIG. 60.

If now we put a second pound of water in the cylinder, it will fill it to twice the height, and, in order to equalise the levels, we shall have to plunge it twice as deep in the larger vessel. When we have done so, the plate will again be held in position by the upthrust, and hence we conclude that at twice the depth the pressure of the water is twice as great. Thus the pressure is proportional to the depth, or **head of water**, as it is called, but does not otherwise depend on the *quantity* of water. It must be clearly understood that this pressure is solely due to the weight of the liquid, and that, as in previous cases, it acts equally in all directions.

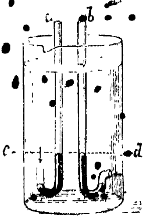

FIG. 61.

That the pressure in a mass of liquid depends on the depth, and that it acts equally in different directions may be demonstrated by taking two glass tubes, bent as shown, each containing some mercury, and plunging them side by side in a deep vessel of water. The deeper they go the farther will the mercury be pushed up by the pressure of the water, and, although in one case the pressure acts downwards and in the other case sideways, it will be observed that the mercury in both tubes will show the same *difference of level* on the two sides of the bend.

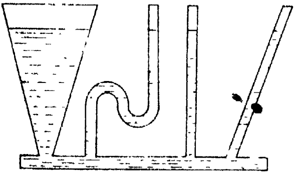

FIG. 62

Another experiment proving that the pressure of water depends simply on the depth and not on the shape or size of the vessel consists in taking a series of different tubes and vessels communicating together, and filling them up with water. It will be observed that the water finds the same level in all the tubes, notwithstanding their differences in size and shape; this, of course, means that the pressure is the same at the base of each tube, other-

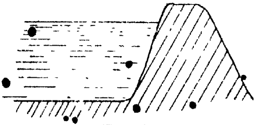

FIG. 63

wise the water would flow towards the point where the pressure was lowest.

The fact that water pressure increases with the depth must be kept in view in the construction of dams, water tanks, dock gates, &c., the greatest strength being necessary where the distance below the surface of the water is greatest. Thus dams are always made wider at the bottom (see Fig. 63).

The Principle of Archimedes.—We are now in a better position to understand the fact, mentioned in chapter iii, that a solid when immersed in a liquid experiences an upthrust, and therefore shows an apparent loss of weight. The diagram represents, let us say, a centimetre cube of a solid with its upper face 1 centimetre below the surface of the water. The arrows show the directions of the water pressure on different faces. Clearly the pressures on the side faces will balance and neutralise each other. The pressure on the top will be equal to the weight of the water lying above it, and, since it is a centimetre square and 1 centimetre below the surface, this will be a cubic centimetre of water, which weighs 1 gram. The pressure on the bottom face, since it is 2 centimetres below the surface, will be 2 grams per square centimetre, and hence equal to 2 grams on the whole face; thus the upward pressure on the lower face exceeds the downward pressure on the top by 1 gram, and since the bottom is always a centimetre lower than the top, the same reasoning will apply at whatever depth the cube be placed, and hence there is always a resultant upthrust and consequent loss of apparent weight of 1 gram; *i.e.* the loss of weight is equal to the weight of water displaced.

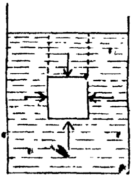

FIG. 64

In the case of a body which is specifically lighter (*i.e.* less dense) than water, it will float, with just so much of the body immersed in the water as will displace *its own weight* of the liquid, so that the upthrust of the water just neutralises the weight of the body.

Example:—

Suppose a 3-inch plank of timber, whose specific gravity is ·4, floats on water, how deep will it sink?

Since the timber is only $\frac{4}{10}$ as heavy as an equal volume of water, it follows that the volume of water whose weight just balances the weight of timber will be $\frac{4}{10}$ the volume of the timber. Hence the floating timber will displace $\frac{4}{10}$ its own volume of water; *i.e.* it will float with $\frac{4}{10}$ of its volume under water and $\frac{6}{10}$ of its volume out of water.

Now $\frac{4}{10}$ of 3 inches = 1·2 inches, which is the depth to which it will sink.

The Hydrometer.—The principle of Archimedes applies to all liquids; thus, a floating body always displaces its own weight of the particular liquid in which it floats; hence it will sink farther in a light liquid than in a heavy one. A hydrometer is simply a float (see Fig. 64A) in which the principle is applied in determining the specific gravities of liquids. The stem is provided with a scale on which the specific gravity of the liquid can be read off according to the depth at which the instrument floats.

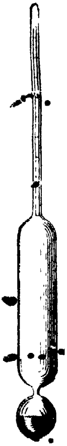

FIG. 64A

The U-tube for Specific Gravities.—If we place two different liquids, which do not mix, in the two limbs of a U-tube, they do not find the same level, because it will require a longer column of the lighter liquid to balance a shorter column of the denser one. If one liquid were twice as dense as the other, it would clearly equire a column only half as long to balance the other; and, in fact, the heights of the two columns will be in the *inverse* ratio of the specific gravities of the liquids. Thus—

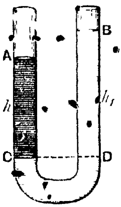

Fig. 65.

$$\frac{\text{specific gravity of liquid A}}{\text{specific gravity of liquid B}} = \frac{\text{height of column of B}}{\text{height of column of A}}.$$

If one of the liquids be water, the specific gravity of the other can thus be readily calculated.

The student should be careful to note that the heavier liquid is usually allowed to pass right round the bend, and the height of its column is measured, *not* from the bottom of the tube, but from its level in the other limb (shown by the dotted line CD).

In the case of two liquids which would mix, it is necessary to place a little mercury in the bend to keep them separate, and the levels of the liquids must be then carefully adjusted so that the mercury stands at *exactly* the same level in both bends, for if there were an excess of mercury on one side the excess pressure would partly balance the liquid on the opposite side.

Summary.

Liquids transmit pressure equally in all directions.

Pressure is proportional to area. The hydraulic press lepends on this.

Pressure due to the weight of a liquid is proportional to :he depth or "head" of water.

Uniform level in various-shaped vessels and tubes.

Principle of Archimedes depends on greater pressure on ower surface; resultant upward force = weight of liquid lisplaced.

Hydrometer—to find specific gravity of liquids: floats deeper in lighter liquids.

Specific gravity by **U**-t

$$\frac{\text{Specific gravity of A}}{\text{Specific gravity of B}} = \frac{\text{height of B}}{\text{height of A}}$$

EXERCISES.

1. In a hydraulic press the small plunger has a diameter of 2·5 centimetres, and the ram a diameter of 30 centimetres; the lever handle has a mechanical advantage of 20. Find the total pressure exerted by the ram when a pressure of 25 kilograms is applied at the end of the handle.

(*N.B.*—The areas of circles are proportional to the *squares* of the diameters.)

2. Find the water pressure in pounds per square inch at the bottom of a tank 20 feet deep.

3. A reservoir is situated at an elevation of 500 feet above a water-main. Find the pressure in pounds per square inch on the main (making no allowance for friction).

4. A rectangular steel pontoon 50 feet × 20 feet × 10 feet weighs 5 tons. What weight will it support without sinking in pure water? How much more would it support in sea-water of specific gravity 1·025?

5. A column of sea-water 10 inches high balances in a **U**-tube a column of paraffin oil $12\frac{1}{2}$ inches high. Taking the specific gravity of sea-water as 1·025, find the specific gravity of the paraffin.

PRACTICAL EXERCISES.

1. Verify the principle of Archimedes by weighing various solid bodies in air and water, and comparing the loss of weight with the weight of water displaced, as found by the overflow-displacement apparatus.

2. Repeat the experiment in Exercise 1, using a liquid

lighter or heavier than water, and observe whether the law still applies

3. Weight a test-tube with shot so that it floats with about $\frac{2}{3}$ of its length under water. Find the weight of the float, and then, by floating in a graduated cylinder, find the volume and the weight of displaced water and compare the two weights.

4. Float the test-tube in Exercise 3 in various heavier and lighter liquids and note the volume of displaced liquid in each case. Find the weight of the same volume of the liquid and observe whether it agrees with the weight of the float.

5. Float a pencil or a long wooden cylinder in water (using guide wires to keep it upright). Measure the length in and out of water, and calculate the specific gravity of the wood.

6. Use a hydrometer to test the specific gravities of various liquids, *e.g.* methylated spirit, paraffin, and solutions of different salts in water.

Compare the specific gravities of strong and weak solutions of salt.

7. Compare the densities of different liquids by balancing columns in a **U**-tube. If the liquids will mix together, mercury must be placed in the bend and the heights carefully adjusted so that the mercury reaches exactly the same level on both sides.

8. Repeat the experiment in Exercise 7 using Hare's apparatus, which consists practically of an inverted **U**-tube dipping into the two liquids and having a branch tube at the top which allows air to be sucked out so that the liquids rise in parallel columns.

CHAPTER XIII

PRESSURE OF GASES

GASES, like liquids, are fluid (*i.e.* they have the power of flowing), and consequently transmit pressure in all directions. Unlike liquids, however gases are compressible and elastic, *i.e* the volume occupied by a given quantity of gas can be varied according to the pressure to which it is subjected.

Pressure of the Atmosphere.—In the previous chapter it has been pointed out that liquids exert pressure on account of their own weight, and that this pressure increases as the depth, or head of liquid, increases. Now, we live at the bottom of what might be termed an ocean of air, which surrounds the earth, and which is known as the atmosphere. In the first chapter we saw that gases, like other forms of matter have weight, and therefore we should expect that the atmosphere would exert pressure, on account of the weight of the air. Under ordinary circumstances we are not conscious of this pressure, because it acts equally in all directions; thus, if we hold out our hands, any downward pressure on the upper surface is just balanced by an equal upward pressure on the lower surface. But, if we remove the air-pressure on one side of a surface, the pressure on the other side at once becomes manifest. Thus, if we cover the mouth of a bell-

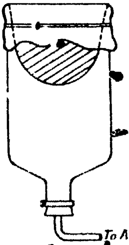

FIG. 66

jar with a sheet of india-rubber, the atmospheric pressure acts equally on the upper and lower surfaces of the latter; but if we remove (or reduce) the pressure from below by pumping air out of the bell-jar, then the pressure on the top pushes the rubber down into a cup-shaped hollow, and may even cause it to burst inwards.

Another experiment may be made with the **Magdeburg Hemispheres** (Fig. 67). Two cups or hemispheres of brass are accurately fitted together to form a hollow sphere or globe. So long as the air remains within, the inner and outer air-pressures balance each other, and we can separate the hemispheres with ease; but if we pump the air out of the hollow globe, then the outside pressure of the air, being now unbalanced, will push the two halves together so forcibly that they can only be separated by a very strong pull.

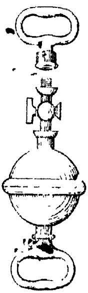

FIG. 67.

The Barometer for Measuring Atmospheric Pressure.—In the last chapter (see Fig. 61) we saw that the increasing pressure, as we go deeper in a vessel of water, may be indicated by a column of mercury pushed higher and higher in a U-tube. The pressure of the atmosphere is also capable of supporting a column of liquid. Thus, if we take a long tube with a tap (say, a burette), and, placing the open end under water, suck out the air at the top, we shall find that, as the air is removed, the water will rise, and will finally fill the tube. The water has, in fact, been pushed up by the pressure of the air acting on the surface of the water in the vessel. If we cover the mouth of the tube with a light card, we may remove the burette from the vessel of water, and the card

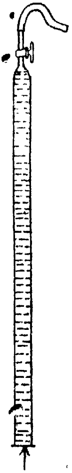

FIG. 68.

will still remain in position. Because, although the pressure of the water is pushing the card downwards, the pressure of the air outside is pushing it upwards more strongly. If we were to make the tube longer and longer, the pressure of the water column would become proportionately greater, and at last we should reach a point at which the pressure of the water neutralises the pressure of the air, and the height of the water column would then measure the pressure of the air. The column of water would have to be about 33 feet high, and it was actually discovered, some centuries ago, that a suction pump would not raise water beyond this height of 33 feet.

By using mercury, which is 13·6 times heavier than water, a column 13·6 times less, viz. about 30 inches, would suffice to balance the atmospheric pressure.

Take a straight tube over 30 inches long, closed at one end, and fill it with mercury. Then, closing the end with the thumb, invert it in a dish of mercury, and it will be found that the mercury in the tube falls, until it stands at a level of about 30 inches above the mercury in the dish. Such an arrangement is called a **barometer**.

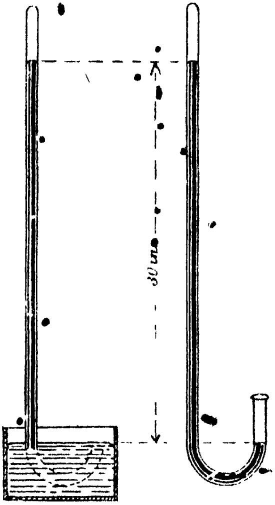

FIG. 60.

Instead of using a separate dish of mercury, we may bend the tube into a **U** shape, when it will be found that the mercury in the closed limb stands at a level about 30 inches above the level in the open limb. We have then a column of mercury about 30 inches high balancing (and therefore measuring) the pressure of a column of air extending to the top of the atmosphere. This form is called a **syphon barometer.**

The closed limb must contain no air at the top, otherwise the atmospheric pressure would be partly neutralised by the pressure of the air inside. If the end of the closed limb were opened or broken, the mercury would fall until it reached the same level on both sides.

The atmospheric pressure varies in different places; thus, if we ascend a mountain, the depth, or head of air, above us becomes less, hence its pressure is smaller, and the mercury in the barometer falls in proportion. But even in one and the same place, the air pressure varies according to the state of the weather, a fact which makes the barometer useful in weather prediction.

Let us now see what is the actual pressure represented by a mercury column of 30 inches. Suppose our barometer tube has a sectional area of just 1 square inch, then, since the volume of a cylinder, whose base is 1 square inch and height 30 inches, is 30 cubic inches, we shall have 30 cubic inches of mercury, and this is found to weigh about 15 lbs. Hence we conclude that the normal or average pressure of the air at sea-level is about 15 lbs. on every square inch of surface.

Boyle's Law.—We are now in a position to measure air-pressures, and to discover the effect of pressure upon volume. Let us take a **U**-tube with a long open limb and a short closed one, and, by putting a little mercury in the bend, let us enclose some air, at atmospheric pressure, in the short limb. We will suppose that we have a column of enclosed air 6 inches long.

If we now pour mercury into the open limb it will exert pressure upon the enclosed air, over and above the pressure of the atmosphere, and we shall find the enclosed air to be compressed into a smaller and smaller volume. When the mercury in the open limb stands 30 inches above the level in the closed limb, the pressure of this mercury will be equal to the pressure of the atmosphere, and will be acting *in addition* to the atmospheric pressure, hence the total pressure on the confined air will now be two atmospheres (equivalent to 60 inches of mercury), and we shall find that the enclosed air has shrunk to a column 3 inches long, *i.e.* by *doubling* the pressure we have *halved* the volume. If the tube be long enough we can add another 30 inches of mercury so as to increase the pressure to three atmospheres, and we shall then find the volume of the enclosed air reduced to 2 inches, *i.e.* to one-third of the original volume.

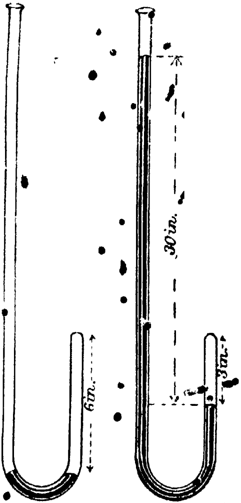

FIG. 70.

In order to discover the effect of diminished pressure, let us take a **U**-tube with equal limbs, one being closed and the other open. First confine a certain volume of air (say, 6 inches) at atmospheric pressure; in order to do this, we must see that the mercury stands at the same level on both sides, so that one column of mercury just balances the other and consequently has no effect on the air. By means of the tap run out mercury until the level in the open limb stands 15 inches *below* that in the closed limb. The mercury tries to find its level, and the column of 15 inches must be supported by the *outside* atmosphere, hence part of this atmospheric pressure (viz. one-half) is used up, as it were, in supporting the mercury, and only the remaining half of the pressure will be effective in pressing on the air inside. Thus the air inside is now only subject to a pressure of half an atmosphere, and its volume will be found to be 12 inches, *i.e.* by *halving* the pressure we have *doubled* the volume; and if we continued until the pressure was reduced to ⅓, the volume would be trebled, and so on.

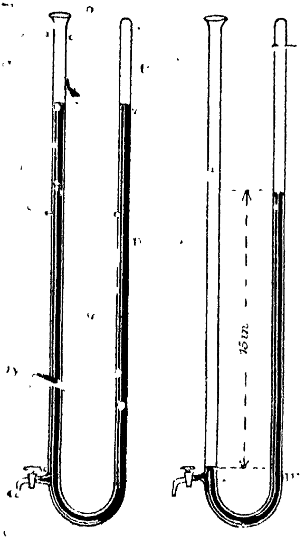

FIG. 71.

Thus it appears that, when we increase or decrease the pressure on the confined air, in any given ratio, the volume of this air undergoes an *opposite* change, in the *inverse* ratio. This is generally expressed as follows:—

Boyle's Law.—The volume of a given mass of any gas varies in inverse proportion to the pressure exerted upon it.

The student should test this important law for himself by making a series of experiments with the "Boyle's tube." Remember that when the mercury is higher in the open limb it is exerting extra pressure on the gas, and the *difference* of levels must be *added* to the height of the barometer, whereas if the mercury be higher in the closed limb it is neutralising part of the pressure of the atmosphere, and the *difference* in levels must be *subtracted* from the height of the barometer.

The results may be tabulated in the following manner :—

No.	Pressures.			Volumes.		
	First.	Second.	Ratio.	First.	Second.	Ratio.
1	30	60	1 : 2	6	3	2 : 1
2	30	90	1 : 3	6	2	3 : 1
3	30	15	2 : 1	6	12	1 : 2
4	30	10	3 : 1	6	18	1 : 3

On comparing the ratios of the pressures and those of the volumes, it will be seen that one can be obtained by inverting the other: hence the quantities are said to be *inversely proportional.* In actual experiments the results will probably not be quite accurate. It should be noted that the true height of the barometer for the time being must be used instead of 30 inches.

This law applies to all gases and even to vapours, such as steam, when in a dry or unsaturated condition; hence the law is of great importance in the theory of the steam-engine.

Examples :—

1. Steam is admitted to a cylinder at a pressure of 25 lbs. per square inch above the atmospheric pressure of 15 lbs. per square inch; at what point of the stroke must it be cut off so that its pressure at the end of the stroke may be 5 lbs. below that of the atmosphere?

The total pressure of the steam on admission = 25 + 15 = 40 lbs. per square inch.

The pressure at end of stroke = 15 − 5 = 10 lbs. per square inch.

Hence the pressure is reduced in the ratio 40 : 10 = 4 : 1.

Therefore the volume must increase in the ratio 1 : 4.

Hence the steam must be cut off at ¼ stroke.

2. 40 cubic feet of coal-gas are forced into a cylinder of ½ cubic foot capacity. Find the pressure of the gas in lbs. per square inch.

The volume is decreased in the ratio of 40 : ½ = 80 : 1.

Hence the pressure must be increased in the ratio of 1 : 80.

But the original atmospheric pressure is 15 lbs. per square inch.

Hence the resulting pressure is 15 × 80 = 1200 lbs. per square inch.

SUMMARY.

Gases are fluid and transmit pressure in all directions. They are compressible and elastic.

Pressure due to weight of atmosphere 15 lbs. per square inch.

Measured by barometer—average height of mercury column, 30 inches.

Boyle's law: volume of gas varies *inversely* as pressure.

EXERCISES.

1. Find the atmospheric pressure on a circular piston 14 inches in diameter, when the barometer stands at the normal height.

2. At the bottom of a deep mine the barometer reads 32 inches. What does this correspond to in lbs. per square inch (taking a barometer reading 30 inches as equivalent to a pressure of 15 lbs. per square inch).

3. A bicycle tyre has a capacity of 1·2 cubic feet when full. How much air must be pumped into it to inflate it to a pressure of 40 lbs. to the square inch?

4. A gasometer contains 100,000 cubic feet of gas when the barometer stands at 29 inches. By how much will this volume be diminished if the barometer rises to 30 inches, assuming that the extra pressure due to the weight of the gasometer corresponds, in both cases to ½ an inch of mercury?

5. Assuming that a litre of air at normal pressure weighs 1·29 grams find the weight of a litre of air on the top of a mountain where the barometer reads 25 inches.

PRACTICAL EXERCISES.

1. Carefully read the height of the barometer in inches and in millimetres.

2. Verify Boyle's law as described in the chapter.

3. On squared paper plot a curve showing the relation between the pressure and volume of a gas, measuring pressures vertically and volumes horizontally. As long a range of pressures as possible should be used, *i.e.* a **U**-tube should be employed having both limbs of a considerable length.

4. By means of a small **U**-tube containing water and attached by rubber tubing to a gas nozzle, determine the gas pressure in the main in inches of water. Calculate the result as the fraction of 1 atmosphere (taking 30 inches of mercury of specific gravity 13·6 as being the normal atmospheric pressure).

CHAPTER XIV

PUMPS AND SYPHONS

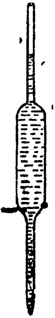

FIG. 7

IN the last chapter we have seen (Fig. 68) that if we suck the air out of a tube which dips in water, the pressure of the outside air pushes the water up the tube, so that it may be filled to any height. If, as in the pipette for measuring liquids, the lower end has only a narrow opening, then if we close the *top* with the finger the tube may be removed from the vessel without the liquid running out of the pipette, being held up by the pressure of the air. As soon as we remove the finger the pressur of the air acts equally on the top and the liquid flows out by its own weight.

Instead of removing the air from a tube by sucking with the mouth, it may be removed by a piston, as in the syringe or squirt. The same principle is applied in the common water pump. In this case there is a kind of trap-door, called a valve, in the piston, which will only open *upwards*, when pushed from *below*; and another valve at the bottom of the pump barrel, also opening only upwards.

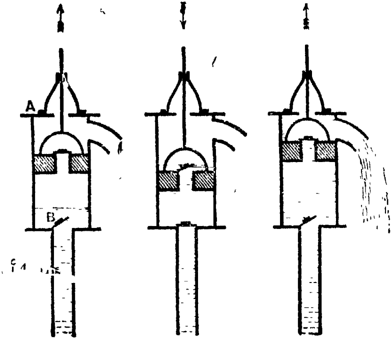

FIG. 73.

Suppose we raise the piston from the bottom of the barrel, it will push out the air lying above it in the barrel, and there will be, below it, a partial vacuum (*i.e.* a space from which the air is partly removed), hence the pressure of the outside air on the water in the well will push it up the connecting tube, and, when it reaches the barrel, it will raise the valve B and enter the barrel, as in the first diagram.

If now, the barrel being nearly full of water, the piston be pushed down, the lower valve at once closes, and prevents the water running back, whereas the valve in the piston is pushed open by the water below and the latter then passes through to the top of the piston as in diagram 2.

Finally, if the piston be again raised, the piston valve will close and prevent the water passing back, hence the piston will raise the water until it overflows at the spout; and at the same time a further supply will be drawn into the barrel through the lower valve (diagram 3) so that the operations may be repeated as often as necessary.

As mentioned in the last chapter, the atmospheric pressure will only sustain a water column 33 feet high, hence a pump of this form will not draw water to a greater height.

The Force Pump.—In this pump the piston is solid, and the second valve is placed at the bottom of the discharge pipe at the side of the barrel (at D).

When the barrel is full of water, and the piston descends, it pushes the water through this valve and up the discharge pipe, so that it may be forced up to any height if sufficient pressure be applied.

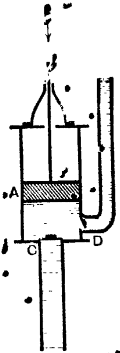

Fig. 74.

A steam boiler feed-pump (not an injector) acts on this principle, and also the pump of a fire-engine; but, in the latter, the water is first forced into an air-chamber, so that the pressure of the confined and compressed air maintains a continuous jet of water, independently of the alternate strokes of the piston.

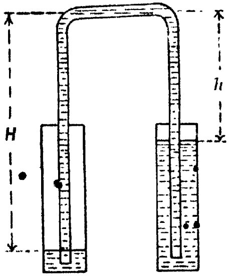

Fig. 75.

The Syphon.—A liquid can be drawn from one vessel to another at a lower level by means of a bent tube, called a syphon. The tube must first be filled with the liquid (by suction or otherwise), and when one end is placed under the liquid in one vessel, it will continue to flow into the other until the levels are equalised, or the first may be emptied to the bottom by making the other leg of the syphon longer.

In the first place, it must be noted that the syphon only remains full of liquid in virtue of the atmospheric pressure acting on the liquid (equally on both sides). If we made a hole in the syphon at the top and allowed the air to act there, the tube would at once empty itself, and the syphon would cease to act.

The air-pressure, being equal on both sides, has, of course, nothing to do with the *flow* of the liquid; this is caused by the fact that one column of liquid (height h) is shorter than the other (height H), and hence the latter pushes downward more strongly, in virtue of the weight of the liquid, and consequently the liquid flows from the shorter to the longer limb. It should be observed, that, provided both legs are below water, the direction of flow depends solely on the water levels in the vessels, and not on the lengths of the two limbs of the tube.

The Bell Syphon.—A form of syphon which is employed in cisterns for flushing purposes is illustrated in two modifications (1 and 2) in the diagram.

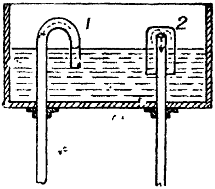

FIG. 76.

Suppose the cistern to be gradually filled from a supply tap; as soon as the water reaches the top of the bend of tube 1, the syphon which it forms will be filled, and will quickly discharge the water in the cistern, and in doing so it will *empty* itself, so that it will not act again until the cistern is refilled. Thus its action is intermittent, and it may be arranged to discharge at any suitable intervals by adjusting the water supply.

In No. 2 in the diagram, instead of the top of the tube being bent round, it is simply covered with a bell; the space between the tube and the bell then acts as the short leg of the syphon. This form is frequently used in W.C.

cisterns, some arrangement being used whereby the syphon can be started by pulling a chain.

It sometimes happens that a syphon of this kind is thrown out of action by the bell becoming corroded and air gaining admission by the small holes thus produced.

SUMMARY.

The air pressure will support a column of liquid, e.g. in a pipette.

The common pump has two valves—one in the suction pipe and one in the piston.

In the force-pump the piston is solid, and the second valve is at the base of the discharge pipe.

In the syphon the flow is caused by the column of liquid in one limb being longer than the other.

In the bell syphon the space between the bell and the pipe forms the short limb of the syphon.

EXERCISES.

1. What horse-power would be required to pump water from a mine 1000 feet deep, at the rate of 50 gallons per minute (making no allowance for loss of power)? One horse-power = 33,000 foot-pounds per minute, and 1 gallon of water weighs 10 lbs.

2. Sketch a suction-pump, a force-pump, a common syphon, and a bell syphon. In the pumps show the position of the valves in both up and down strokes.

CHAPTER XV

HEAT AND TEMPERATURE.

A KETTLEFUL of water upon the fire becomes hotter, as heat is imparted to it by the fire. When we wish to refer to its degree of hotness, we speak of its "temperature," and the more heat the water receives the higher its temperature is raised. Thus, we might think that the temperature of the water is the same as the quantity of heat it contains; but a moment's reflection will show that the two things are quite different; thus, if we remove a cupful of the water, it will be at the same temperature as the water in the kettle, since both are equally hot, but it is clear that this smaller quantity of water will not contain the same quantity of heat as the larger quantity in the kettle. We must distinguish, then, between the temperature of a body and the quantity of heat it contains. The quantity of heat *depends* on the temperature, but it also depends on the quantity of material, and, as we shall see later, also on the kind of material.

By temperature we mean the degree of hotness, and, when we cause an object to become hotter, we are said to "raise" its temperature. There is, in fact, a certain analogy between temperature and water level, for just as water tends to flow from a higher to a lower level (irrespective of the *quantity* of water), so heat tends to pass from bodies at a higher to those of a lower temperature, until the temperature, or heat level, becomes equalised.

The sense of touch enables us to distinguish broadly between hot and cold bodies, but this sense may often be misleading. If one touches a piece of hot metal and a piece of wood at the same temperature, the metal feels hotter than the wood; whereas if both the metal and the wood are colder than the hand, the metal feels colder than the wood. This is because the metal, being a good conductor of heat, conveys heat to or from the hand more readily than the wood. Again, if one hand happens to be warm and the other cold, and both are plunged into water at a temperature between the two, the water will feel warm to the cold hand, and cold to the warm hand. Clearly, then, we must have some more reliable method of estimating temperature than the sense of *touch*.

Fortunately, it happens that change of temperature is accompanied by another change which is readily measurable, viz. a change of volume.

If we fill a bottle quite full of water and then heat it, it will be found that the water appears to expand or swell in bulk, and it will eventually overflow. The change can be more readily seen if we close the bottle with a stopper carrying a narrow tube, the water which is pushed out of the bottle by the expansion will then pass up the tube, and, since the latter is narrow, even a small increase of volume will cause a considerable rise in the level (Fig. 77).

It will then be seen that the more the water is heated the more it will rise in the tube, thus the amount of rise will serve to indicate how much the temperature of the water has been raised. On this simple principle is based the instrument for measuring temperature, called a **thermometer**. It has been found that water does not expand uniformly, and hence another substance, the liquid metal, mercury, or quicksilver, is generally employed. For the bottle we substitute a small bulb (spherical or cylindrical) to which is attached a tube with a fine, hair-like bore. The bore is made narrow in order that even a very small increase in the volume of the mercury may cause an appreciable rise in the stem.

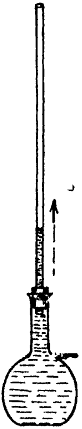
FIG. 77.

In order that such an instrument may serve to *measure* changes of temperature, it must evidently have a scale, based upon some definite unit. The first step in providing such a scale is to mark *two* fixed temperatures, just as the standard yard is the distance between *two* fixed marks.

Now, it has been found that the temperature at which pure ice melts, and the temperature at which pure water boils (under a constant atmospheric pressure) are fixed and invariable temperatures, and therefore very suitable for determining our thermometric scale. The difference in temperature between these two fixed points will clearly be a fixed amount, and if we divide this interval into a number of equal steps or "degrees," each degree will represent a definite interval of temperature, and will therefore be suitable for a unit of measurement.

There are two "scales" in common use, viz. the **Fahrenheit** scale, as used for general purposes in Great Britain, and the **Centigrade** scale, which is mostly used in scientific work. Both are based upon the two fixed points named above, but they are numbered differently, as shown in the diagram.

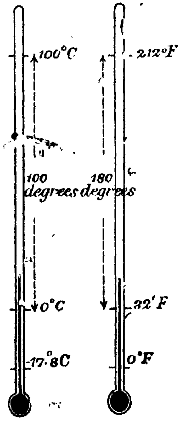

Fig. 78

It will be observed that, whereas, on the Centigrade scale the freezing point is marked 0° C., the corresponding point on the Fahrenheit scale is marked 32° F., and consequently the Fahrenheit zero (0° F.) represents a temperature below the freezing point and hence also below the Centigrade zero.

This Fahrenheit zero corresponds to what Fahrenheit supposed to be the lowest temperature attainable, which he therefore took as the starting point of his scale. We now know that much lower temperatures can be reached; these have to be recorded as so many degrees below zero, and in writing are distinguished by the minus sign (−). Thus −10° F. means 10° below the Fahrenheit zero, and therefore 32 + 10, *i.e.* 42° F. below the freezing point. −20° F. is a temperature 10° *lower* still, *i.e.* the numbers now increase as we go down the scale.

Conversion of Scales.—In converting a temperature from one scale to the other, the following two points must be kept in mind:—

1. The **Fahrenheit scale has**, as it were, **a start of 32° at the freezing point.**

2. The Fahrenheit degrees are shorter, in the ratio of 5 : 9. Thus the interval between freezing and boiling points is divided into 100 Centigrade degrees and 180 Fahrenheit degrees.

Hence 100 Centigrade degrees are equivalent to 180 Fahrenheit degrees, *i.e.* **5 Centigrade degrees = 9 Fahrenheit degrees.**

(*N.B.*—This does *not* mean that 100° C., *i.e.* the boiling point = 180° F., which is not the boiling point.)

Examples:—

1. To convert 20° C. to the Fahrenheit scale.

Since 5 Centigrade degrees = 9 Fahrenheit degrees, 20 Centigrade degrees = $\frac{20}{5} \times 9 = 36$ Fahrenheit degrees (*above freezing point*).

Hence Fahrenheit temperature = 36 + 32° = **68° F.**

2. To convert 59° F. to Centigrade.

59° F. is 59 − 32 = 27 Fahrenheit degrees above freezing point.

But 27 Fahrenheit degrees = $\frac{27}{9} \times 5 = 15$ Centigrade degrees above freezing = **15° C.**

3. To convert −20° C. to Fahrenheit.

This is 20 Centigrade degrees below freezing point.

But 20 Centigrade degrees = $\frac{20}{5} \times 9 = 36$ Fahrenheit degrees (*below* freezing point).

But, as the freezing point is 32° F., a temperature 36° below freezing will be 32 − 36 = **−4° F.**

4. To convert −20° F. to Centigrade.

−20° F. is 20 degrees below *zero* Fahrenheit.

Hence it is 20 + 32 = 52 Fahrenheit degrees below freezing.

But 52 Fahrenheit degrees = $\frac{52}{9} \times 5 = \frac{260}{9} = 28{\cdot}9$ Centigrade degrees.

And 28·9 Centigrade degrees below freezing is **−28·9° C.**

The chief point to be remembered is that the 32° must always be added to or subtracted from Fahrenheit numbers and never from the numbers when they are on the Centigrade scale.

SUMMARY.

1. Temperature is degree of hotness. Quantity of heat depends on other things beside temperature.

2. A thermometer measures temperature by the expansion of a liquid (generally mercury).

3. Freezing point (0° Centigrade and 32° Fahrenheit) and boiling point (100° C. and 212° F.) are fixed points.

4. To convert Centigrade to Fahrenheit, multiply by $\frac{9}{5}$ and add 32 (last).

To convert Fahrenheit to Centigrade, subtract 32 (first), then multiply by $\frac{5}{9}$.

EXERCISES.

1. The temperature of a certain furnace when white-hot was found to be 1300° C. What is this on the Fahrenheit scale?

2. Iron melts at about 1500° C., lead at 325° tin at 235° C. What are the corresponding Fahrenheit temperatures?

3. A mixture of snow and salt has a temperature of about -18° C. Is this above or below the Fahrenheit zero, and by how much?

4. Two rooms are found to differ in temperature by 12° F. How many degrees would this be on the Centigrade scale? (Think carefully before answering.)

5. 60° F. is a comfortable temperature for a living room. What would this be on the Centigrade scale?

6. The steam in a locomotive boiler (being under pressure) is found in a certain case to be at 350° F. Convert this to the Centigrade scale.

PRACTICAL EXERCISES

1. Examine a thermometer and find out whether it is Centigrade or Fahrenheit. Place it first in melting ice and then in the steam from boiling water and record any errors you may find in the fixed points. Read the barometer to see if the pressure is normal.

2. Place the thermometer (*a*) in a mixture of pounded ice and salt, and (*b*) in some boiling salt and water. What has been the effect of the salt on the freezing and boiling points respectively?

3. If possible, compare a Fahrenheit and a Centigrade thermometer by placing them, side by side, in a small vessel of water at different temperatures, recording the results in parallel columns, and then by calculation converting the numbers in one scale to the other.

CHAPTER XVI

EXPANSION BY HEAT

In the last chapter it has been pointed out that, when water and other liquids are heated, they expand or increase

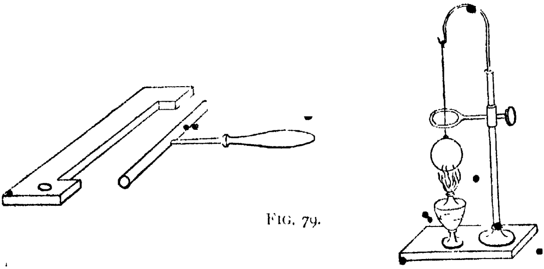

FIG. 79.

in volume. A few experiments will show that solids also become larger when heated; but the amount of expansion is so small that it requires some care to detect it.

If we take a metal bar and a gauge made to fit it exactly when cold, it will be found that when the bar is heated it becomes slightly longer, and will not fit the gauge. It also becomes slightly thicker, and will not fit the small hole into which it will pass freely when cold.

Or we may use a metal ball with a ring whose size is carefully adjusted to fit it. When the ball is hot it will no longer pass through the ring.

In both cases it will be found that the metal regains its original size when it becomes cold.

Again, if a fine iron wire, some yards long, be tightly stretched, it will be found that if we make the wire red hot by means of an electric current it will become slack, and sag perceptibly on account of the expansion.

In order to measure the actual amount of expansion, some method must be employed to magnify the effect, or we must use some specially refined method of measurement, *e.g.* the micrometer screw-gauge.

Coefficient of Expansion.—A very simple [illegible] of measuring the expansion of a metal wire is based upon the magnification of range of motion by a lever of the third order.

A wire AB, say, 4 or 5 feet long, is hung securely from a nail in the wall at A, and the lower end is attached at B to a lever CD, hinged at C, the end D being opposite a vertical scale. The wire is surrounded by a glass tube, through which a current of steam can be passed so as to heat the wire to a *definite* temperature (100° C.). The wire, when heated, becomes slightly longer, and the end B drops, say, to B^1, and allows the lever to fall with it. The distance BB^1 is very small, but clearly the other end D of the lever will move over the scale through a greater distance. In fact, the distance DD^1 will be exactly as many times greater than BB^1 as the total length of the lever CD is longer than the short arm CB. Thus, the expansion is, as it were, magnified in a definite proportion. For instance, if the lever is 90 centimetres long, and CB is 3 centimetres long, the distance DD' (as indicated on the scale) will be just 30 times the expansion BB'.

The amount of expansion will evidently depend on how long the wire is; thus, a wire 2 metres long will expand twice as much as one 1 metre long; and again, it will depend on the rise of temperature; thus, a rise of 20° will cause twice as much expansion as a rise of 10°.

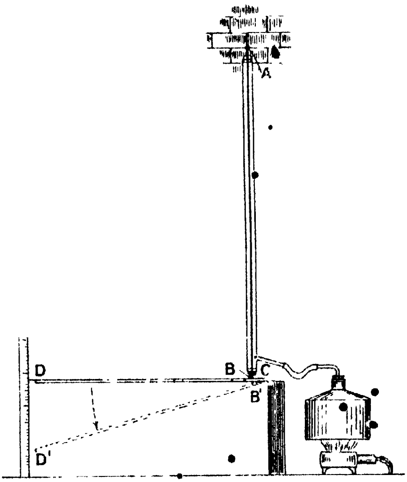

FIG. 80.

The usual method is to state the expansion of *unit length* of the solid when heated 1° C. It should be noted that it does not matter in the least which unit of length we use; thus if a rod a yard long expands $\frac{1}{100000}$ of a yard, then a rod an inch long would expand $\frac{1}{100000}$ of an inch, and one a centimetre long $\frac{1}{100000}$ of a centimetre; in fact, a rod of *any* length will expand $\frac{1}{100000}$ of its own length.

The fraction which represents **the expansion of unit length for a rise of one degree** in temperature is called the **coefficient of expansion,** or since it represents the increase in *length* it may be called the coefficient of *linear* expansion.

When stated in this way we can readily compare the expansion of one substance with that of another.

Example:—

A wire, 2 metres long, is heated from 10° C. to 100° C., and the expansion, when magnified 30 times by the lever, is represented by 9·3 centimetres on the scale. We are required to find the coefficient of expansion.

The actual expansion will be $\frac{1}{30}$ of 9·3 = ·31 centimetres.

This is the expansion for a rise of 90° C.

Hence the expansion for 1° C. is $\frac{1}{90}$ of ·31 = ·00344 centimetre.

But this is the expansion of a wire 2 metres = 200 centimetres long.

Hence the expansion for 1 centimetre = $\frac{1}{200}$ of ·00344
= ·0000172.

This is the coefficient of expansion of the metal. All the steps in the working could be combined in a single formula thus: Coefficient = $\frac{1}{30}$ of 9·3 × $\frac{1}{200}$ × $\frac{1}{90}$ = ·0000172.

In fact we may give the general formula:—

$$\text{Coefficient} = \frac{\text{actual expansion}}{\text{length} \times \text{number of degrees rise of temperature}}.$$

The coefficient of expansion of a particular kind of material may be found experimentally, once for all, and we shall then be able to calculate the actual expansion of a piece of the solid of any size when heated any given number of degrees.

The following are the coefficients of some common substances:—

Iron and steel = ·000011.
Brass = ·000019.
Copper = ·000017.

Example :—

An iron girder is 30 yards long. How much will it expand when heated from the freezing point to 25° C.?

An iron rod 1 inch long heated 1° C. would expand ·000011 inches.

Hence the girder being 30 × 36 inches long would expand for 1° rise of temp. } ·000011 × 30 × 36 inches.

Therefore when heated 25° it would lengthen by

·000011 × 30 × 36 × 25 = ·297 inches, *i.e.* rather more than ¼ inch.

This example serves to illustrate the fact that the amount of expansion which takes place is extremely small; but, on the other hand, it must be remembered that it takes place with enormous force, so that, if the natural expansion and contraction of a metal structure be resisted, the metal may be bent or broken.

For instance, it has been found by actual test that an iron rod 30 yards long and 1 square inch in section would require a force or pull of 3 tons to stretch it by ¼ inch; hence we may conclude that if such a rod were expanded by heat to this extent (as in the last example) and then cooled without being allowed to contract again, it would exert a force of contraction of 3 tons, and a thicker rod would exert a proportionately greater force; the force of expansion, if resisted, would be equally great.

Hence provision for free expansion and contraction must be made in engineering work. Thus the rails on a railway are laid with small spaces between them, and the chairs and fishplates are so arranged that expansion and contraction can take place freely. Again, the long girders used in bridge construction rest upon the piers or supports in such a manner that the ends can move freely backwards and forwards to provide for the slight variations in length. Telegraph wires if stretched too tightly in summer are liable to snap when subjected to the winter's cold, especially when weighted with a burden of snow.

In some cases the great force of contraction is usefully applied; *e.g.* the iron tyres on wooden cart-wheels and the tyres on some railway waggon and carriage wheels are put on when hot, so that the shrinkage on cooling binds the whole firmly together.

The rivets in boiler plates and other iron work are put in red hot, so that the shrinkage which occurs after the rivet head has been made draws the plates firmly together.

The cracking of a thick glass vessel when hot water is poured into it is caused by *unequal* expansion, the part next the hot water tending to expand more than the colder part outside. A thin glass vessel does not crack in this way because, being thin, the heat rapidly penetrates and the whole expands uniformly. The tendency to crack may be lessened by careful annealing, *i.e.* by heating the glass to a high temperature and letting it cool very slowly.

The rate of ordinary clocks and watches is affected by temperature. Thus the rate of a clock depends on the length of the pendulum, and if the pendulum becomes slightly longer by expansion the clock will go slow. In the best clocks and watches this tendency is counteracted by an automatic compensating arrangement.

Expansion of Liquids.—As we have seen in the last chapter, the expansion of water and other liquids is very easily detected by completely filling a flask or bulb with the liquid and attaching a long narrow stem up which the liquid is forced by the expansion. If such a flask be *suddenly* heated, by placing it in very hot water, it will be observed that there is a momentary *fall* of the liquid in the stem, followed by a gradual rise; the final rise being much greater than the original slight fall. In order to understand this, we must remember that the heat first reaches the glass vessel, which therefore expands slightly, before the liquid has time to get hot, and since it thus makes more room in the vessel some of the liquid in the stem passes down to take up this extra space. The fact that the liquid finally rises higher than the original level shows that the liquid in the end expands more than the glass; the amount of rise, in fact, indicates the *difference* between the expansion of the liquid and that of the glass; this is called the **apparent expansion.** In order to find the true or **absolute expansion** of the liquid, we must add on the expansion of the solid.

The **Coefficient of Expansion** of a liquid is the **increase in volume** of **unit volume** for a rise in temperature of **1° C.**

Thus we may use the formula:—

$$\text{Coefficient of expansion} = \frac{\text{increase in volume}}{\text{original volume} \times \text{No. of degrees rise in temperature}}.$$

In order to find the original volume, we must know the volume of the flask or bulb, and in order to find the increase of volume, we must know the volume of a given length of the stem.

Example:—

Suppose that the flask has a volume of 100 c.c., and that on heating from 10° to 50° the liquid rises 10 centimetres in the stem. Suppose also that previous experiment shows the *volume* corresponding to 1 centimetre *length* of the stem is ·25 c.c.

The actual expansion (corresponding to 10 c.m. of stem) = ·25 × 10 = 2·5 c.c.

$$\text{Hence expansion of 1 c.c. for 1° C.} = \frac{2·5}{100 \times 40} = ·000625.$$

This represents the coefficient of apparent expansion of the liquid.

It is an easy matter to compare the expansion of different liquids by placing them in flasks of the same size provided with stems of equal bore, putting all the flasks together into the same vessel of hot water. The liquid with the greatest coefficient of expansion will, of course, rise farthest up the stem.

An important consequence of the expansion of liquids is that the hot liquid has a smaller density than the cold liquid, because there is a smaller mass of liquid in a given space, and hence the warm liquid tends to float on the top of the cold liquid. This will be referred to again in chapter xx.

It has already been pointed out that water does not expand uniformly, thus a given quantity of water expands more between 50° and 60° than it does between 10° and 20°, or, in other words, the coefficient of expansion is greater at higher than at lower temperatures.

Still more remarkable is the fact that below 4° C. water actually expands on cooling, and contracts on heating. Thus, a given quantity of water occupies a smaller volume at this temperature than at any other (whether higher or lower), it is therefore known as the temperature of *maximum density*

When the water in a pond is cooled from the top, the cold water, being more dense, sinks to the bottom, until all the water in the pond reaches 4° C., after this point the still colder water, being now less dense, remains at the top, and hence the freezing of the water commences from the top, and not from the bottom, as would have been the case had the water continued to contract right down to the freezing point.

Expansion of Gases.—If we take a bladder partly filled with air and place it near a fire, it swells up on account of the expansion of the air inside. Air and other gases are, in fact, very sensitive to changes of temperature, a very slight rise of temperature producing a very noticeable increase of volume.

Thus, if we take a small bulb with a narrow stem and put the open end under water, on placing the warm hand on the bulb some of the air is driven out, and can be seen passing in bubbles through the water. On removing the source of heat, the remaining air contracts again, and the water rises in the stem to take the place of the air which has been expelled. On warming it again, the column of water is driven down. (See Fig. 81.)

The **differential air thermometer** is used for detecting slight *differences* in temperature; it consists of two bulbs, connected by a bent tube, containing liquid in the bend.

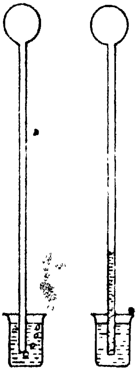

FIG. 81

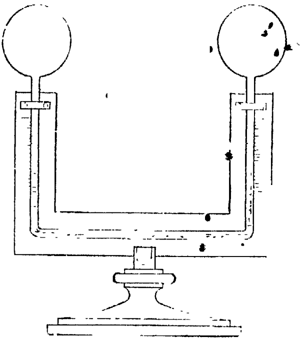

FIG. 82

If one bulb be warmed more than the other, the expansion of the air drives the liquid towards the colder bulb.

If air be heated in a closed vessel, so that it cannot expand, the effect is to increase its pressure. In accordance with Boyle's law, the increase of pressure is in exactly the same proportion as the increase in volume would have been had the gas been allowed to expand freely.

We have seen that different solids and liquids have different coefficients of expansion; in the case of gases the matter is different, as it has been found that all gases have practically the same coefficient of expansion. This coefficient may be expressed as the decimal fraction, ·00366, or as the vulgar fraction, $\frac{1}{273}$. This means that if we have any volume of gas at 0° C. it will expand by $\frac{1}{273}$ of its volume at 0° for every degree rise of temperature, so that at 273° its original volume would be doubled.

If we started with 273 c.c. of gas at 0° C. it would expand thus:—

273 c.c. at 0° C.
273 + 1 = 274 c.c. at 1° C.
273 + 10 = 283 c.c. at 10° C.
273 + 100 = 373 c.c. at 100° C.
273 + 273 = 546 c.c. at 273° C.

It would also contract at an equal rate below 0° C., and hence it would become:—

272 − 1 = 272 c.c. at −1° C.
273 − 10 = 263 c.c. at −10° C.
273 − 100 = 173 c.c. at −100° C.

At −273° C., if the same rule held good, its volume would be reduced to 273 − 273 = 0.

This temperature of −273° C., at which the volume would become 0, if the gas continued to contract at the same rate, is now regarded as the real zero or **absolute zero of temperature**, *i.e.* it corresponds to an entire absence of heat, and consequently no lower temperature is possible. All gases, in point of fact, become liquid before this temperature is reached and so cease to obey the rule.

If we wish to reckon temperatures from the absolute zero, mentioned above, since this is 273 degrees lower than the Centigrade zero it follows that we must *add* 273 degrees to each Centigrade temperature in order to get the corresponding **absolute temperature.**

If the temperature in the preceding table be thus converted to the absolute scale it will be found that the volumes of the gas at different temperatures are proportional to the number of degrees on the absolute scale

This is a most convenient rule for calculating the change in volume of a gas due to change of temperature.

Example:—

Suppose we have 100 c.c. of air at 15° C., what will its volume become at 50° C. and at 0° C.?

$15°$ C. = 273 + 15 = 288 degrees on the absolute scale.
$50°$ C. = 273 + 50 = 323 „
$0°$ C. = 273 + 0 = 273 „

Hence when heated from 15° C. to 50° C. the gas expands in the ratio 323 to 288 or $\frac{323}{288}$.

Since the volume at 15° is 100

The volume at 50° is $100 \times \frac{323}{288} = 112{\cdot}1$ c.c.

When cooled from 15° C. to 0° C. the gas contracts in the ratio of 273 to 288 or $\frac{273}{288}$.

Hence volume at 0° C. = $100 \times \frac{273}{288} = 94{\cdot}8$ c.c.

As in the case of liquids the expansion of gases makes them less dense, hence heated air always tends to rise. This is the cause of the upward draught in a chimney which is so necessary to draw in the air required for the burning of the fire. The taller the chimney and the hotter the air in it the more powerful will be the draught.

Some systems for the ventilation of buildings depend on the tendency of heated air to rise; outlets are frequently provided near the roof for the escape of the hot, vitiated air.

Winds are, generally speaking, produced by the unequal heating of the atmosphere, the warmer air rises and the cooler air from a distance flows in to take its place, thus producing wind, which is simply a current of air or draught on a large scale.

SUMMARY.

1. Solids, liquids, and gases all expand on heating.
2. Coefficient of expansion

$$= \frac{\text{Actual expansion in volume (or length)}}{\text{Original volume (or length)} \times \text{No. of degrees rise}}$$

Hence actual expansion = coefficient × rise × total volume (or length).

3. Enormous force of expansion and contraction (hence allowance in rails and bridges).

4. Gases expand more than solids and liquids, and all have the same coefficient, viz. $\frac{1}{273}$ or ·00365.

5. $-273°$ C. is called *absolute* zero.

6. Volume of a gas proportional to *absolute* temperature (= Centigrade temperature + 273°).

Exercises.

1. What would be the total expansion in inches of a mile of railway line for a rise of temperature of 20° C.?

2. What amount of play should be allowed in an iron girder 50 feet long to allow for a range of temperature from 0° to 100° *Fahrenheit?*

3. A brass yard measure is accurately graduated at 15° C. What would be its true length at 0° C. and at 100° C.?

4. By how much would a litre of air expand when heated from 0° to 20° C.?

5. A room contains 150 cubic feet of air. How much would be expelled when the temperature rises from 15° C. to 20° C.?

Practical Exercises.

1. Support a rod of iron (preferably 18 to 24 inches long) on wooden blocks or boxes at each end. Put a heavy weight on one end, and let the other end rest on a fine needle, so that, when the rod expands on heating with a gas burner, it rolls the needle along. In order to make the slight motion of the needle clearly visible, stick the point through the middle of a light bit of straw to act as an index. Try the effect of heating and cooling the rod.

2. If a piece of iron gas-pipe 2 or 3 yards long can be

obtained, let it rest on two bits of glass tubing, with one end placed against a heavy weight and the other opposite a fixed millimetre scale. On passing a current of steam from a small flask or boiler through the tube, a measurable expansion will be obtained. It will be convenient to use a piece of wire twisted tightly round the end of the tube and projecting over the scale, as an index.

From the results the coefficient of expansion of the iron can be calculated if the temperature of the iron when cold be taken by slipping a thermometer inside it.

3. Compare the expansion of water and alcohol by taking two equal flasks (say, 2 oz.), fitting them with tight stoppers carrying narrow glass tubes of the same diameter, and after filling with the liquids, placing them side by side in a pan of water whose temperature is kept steadily, say, at 50° C., until expansion ceases (which will take some time). If the capacity of the flasks and the calibre of the tubes be found, the coefficients of expansion can be calculated.

4. Weigh a small stoppered bottle (*e.g* a specific gravity bottle) full of cold water, then heat it up to 50° C. by placing it in a vessel of water kept at that temperature until the water inside reaches that point (it may be tested by removing the stopper and inserting a thermometer, but do not let the stopper become cold before putting it in, otherwise it will expand and stick in the neck). Then remove the bottle, dry it, and weigh again. Calculate what fraction of the water has been expelled, and dividing by the number of degrees rise of temperature, find the coefficient of expansion.

Strictly speaking, the formula should be:—

$$\text{Coefficient of expansion} = \frac{\text{weight expelled}}{\text{weight } \textit{left} \times \text{range of temp.}}$$

5. Obtain a piece of barometer tubing of narrow bore, sealed at one end, and containing some air confined by

a short column of strong sulphuric acid. Measure the length of the air column at various temperatures (obtained by placing the tube in water at the required temperature, but with the mouth of the tube out of water), and plot the results on squared paper. If the expansion is uniform, the "graph" should be a straight line. By producing the line backwards the volume of the air at 0° C. is found, and the coefficient of expansion may be calculated.

N.B.—Do not jar the tube, or the sulphuric acid column may break up.

CHAPTER XVII

CALORIMETRY, OR MEASUREMENT OF HEAT QUANTITIES

If a hot metal ball be dropped into cold water, the water will become hotter and the ball colder. Something, which we call heat, has passed from the ball to the water. If we weigh the ball carefully when hot and again when cold, its weight will be found to be unchanged. Thus heat is not a material substance, and it cannot be measured by weighing.

If we run 1000 cubic feet of water into a reservoir whose area is 1000 square feet, the water-level will be raised by 1 foot, whereas if we run this 1000 cubic feet into another reservoir whose area is only 500 square feet, the level will be raised by 2 feet.

Thus, in estimating the quantity of water in a reservoir we must take account both of the area or capacity of the reservoir and of the depth or level of the water. In a similar manner, if we wish to know how much heat we can get out of a body, we must take account of its heat capacity and also of its temperature or heat level. Now, the heat capacity, as we shall see later, depends both on the weight of the body and on the kind of material.

Let us consider some preliminary experiments with hot and cold water, which can be readily mixed so as to reach a uniform temperature.

First suppose we mix equal weights (say, 100 grams of each) of cold water at 10° C. and warm water at 50° C.; the resulting mixture will be found to be at 30° C.[1] Thus the warm water in cooling 20° C. has just furnished sufficient heat to raise the cold water 20° C. Again, if we mix, say, 100 grams of hot water at 50° C. with 300 grams of cold water at 10° C., the resulting temperature will be 20° C., *i.e.* the 100 grams of hot water in cooling 30° C. has only yielded heat sufficient to heat the 300 grams of cold water 10° C.

It will be observed that the number of grams of hot water multiplied by the number of degrees its temperature falls is equal to the number of grams of cold water multiplied by the number of degrees its temperature rises. Thus it appears that the quantity of heat given or received by a mass of water will be proportional to the weight of water multiplied by the rise or fall of temperature in degrees.

It is therefore convenient to choose the following as our unit of heat: the **thermal unit or calorie** *is the quantity of heat given out or received by one gram of water when its temperature rises or falls by 1° C.*

The British thermal unit *is the quantity of heat required to heat one pound of water 1° Fahrenheit,* and is often used in engineering work.

Let us now consider an experiment in which we mix 100 grams of iron tacks heated, by surrounding with steam, to 100° C. with 100 grams of water at 10° C. The resulting temperature will be found to be *about* 19° C. Thus the iron in cooling 81° C. has only yielded sufficient heat to raise an equal weight of water 9° (*i.e.* only $\frac{1}{9}$ as many degrees). It thus appears that the heat capacity of the iron is only about $\frac{1}{9}$ or ·11 of that of an equal weight of water.

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[1] The vessel which contains the mixture will absorb some of the heat and interfere with the accuracy of the result, but this can be overcome either by making the mixture first in the hot vessel and transferring it to the cold one, or by making an allowance for the heat capacity of the vessel. If this be of copper or brass, it may be taken as practically $\frac{1}{10}$ its weight of water. Thus if it weighs 80 grams, it will absorb as much heat as 8 grams of water, and hence we use 8 grams less of the cold water.

In a similar way it might be found that the heat capacity of lead is only about $\frac{1}{32}$ or ·031 of that of water.

These numbers which indicate the heat capacities of different substances as compared with the *same weight* of water are called the **specific heats** of the substances.

We may state briefly that for any substance the **specific heat**

$$= \frac{\text{quantity of heat required to change given weight of substance } 1^\circ \text{ C.}}{\text{quantity of heat required to change given weight of water } 1^\circ \text{ C.}}.$$

In the above experiment for finding the specific heat of the iron tacks it will be observed that the excess heat of the iron was transferred to *a known weight of water*, which acted as a heat reservoir *of known capacity*; and, by the rise in temperature produced, the quantity of heat given up by the iron was estimated. The measurement of heat quantities is called calorimetry and the vessel containing the weighed quantity of water is known as a *calorimeter*.

A very simple experiment to show that iron has a higher specific heat than lead is to take equal weights of iron tacks and lead shot in separate test-tubes and place them in a vessel of boiling water until they reach 100° C. Then quickly transfer the metals to equal weights of cold water, at the same temperature, contained in two calorimeters, and test the final temperatures by means of a differential air thermometer. The water heated by the iron will be found hotter than that heated by the lead.

Temperature of a Furnace by Calorimetry.—An interesting application of the calorimeter is to find the emperature of a furnace by using a piece of metal whose pecific heat is known.

Suppose, for instance, we take an iron rod weighing 50 grams and place it in the furnace until it reaches the ame temperature, and then transfer it to a calorimeter ontaining 300 grams of water at a known temperature— ay 10° C. The water is then well stirred until the final emperature is uniform, and we will suppose this to be 35° C., *i.e.* the water has been heated 25° C.

The number of heat units received by the water is $300 \times 25 = 7500$. This heat must have come from the heated iron.

Now 50 grams of water in cooling 1° C. would give out 50 units of heat. But since the specific heat of iron is ·112, the 50 grams of iron will give out $50 \times \cdot112$ units $= 5\cdot6$ units in cooling 1° C.

We have now to find how many degrees the iron must cool to yield the 7500 units which were received by the water, *i.e.* we must find how many times 5·6 is contained in 7500, hence :—

The number of degrees the iron has cooled $= \dfrac{7500}{5\cdot6} = 1339°$ C.

But as the final temperature of the iron was 35° C., the original temperature (*i.e.* temperature of furnace) $= 1339 + 35 = 1374°$ C.

The High Specific Heat of Water.—As water is the standard of comparison for specific heats, its specific heat is, of course, 1, whereas the specific heats of practically all other substances are represented by fractional numbers, less than 1. Thus, water has a greater heat capacity than other substances, hence it requires more heat to raise its temperature, and when it cools it gives out more heat than a corresponding weight of any other substance.

This fact has certain important practical consequences. Water is a very effective medium for storing and distributing heat, and is used for this purpose in heating buildings by hot-water pipes and in railway carriage foot-warmers. In the use of water in the condensers of steam-engines we take advantage of the same property.

It has also an important influence upon climate, thus the temperature of the sea rises and falls much more slowly than that of the land, because a larger quantity of heat must be added to, or removed from, the water to produce a corresponding change of temperature. On this account, places near the ocean are not subject to the same extremes of temperature as are found to exist in the interior of a large continent.

SUMMARY.

1. Quantity of heat in a body depends on (*a*) temperature, (*b*) quantity of material, (*c*) kind of material.

2. *Thermal Unit.*—The quantity of heat required to heat 1 gram of water 1° C.

British Thermal Unit.—Quantity of heat required to heat 1 lb. of water 1° F.

3. *Specific Heat*

$$= \frac{\text{quantity of heat required to heat given weight of substance } 1^\circ \text{ C.}}{\text{quantity of heat required to heat same weight of water } 1^\circ \text{ C.}}$$

4. Process of finding heat quantities called calorimetry.

5. Temperature of furnace by finding quantity of heat given out by known weight of metal heated in the furnace.

6. Water has a higher specific heat than other substances. This is important in heating buildings, and in its effect on climate.

EXERCISES.

1. How many British thermal units will a gallon of water (10 lbs.) give out in cooling from 200° F. to 60° F.?

2. How many calories will be given out by a kilogram of water in cooling from 100° C. to 15° C.?

3. Taking the specific heat of iron as ·112, find how many degrees a litre of water (1000 grams) would be raised by the heat given out by half a kilogram of iron in cooling from 100° C. to 20° C.

4 A copper calorimeter weighs 100 grams; taking the specific heat of copper as ·095, find the weight of water which would require the same quantity of heat for every degree its temperature is raised, as the copper vessel requires.

(NOTE.—This weight is called the "water equivalent" of the calorimeter.)

5. If the burning of 1 lb. of coal yields 13,000 British thermal units find how many gallons of water would be heated from 60° F. to 200° F. by burning 5 lbs. of this coal.

6. Being told that the condensation of 1 lb. of exhaust steam, and cooling the resulting water to 100° F., liberates 1056 British thermal units, find how many gallons of condensing water at 60° must be allowed for the condensation of each pound of steam, so that the temperature of the water may not be raised above 100° F.

7. An iron ball weighing 1 lb. is heated to the temperature of a furnace, and plunged into half a gallon of water at 50° F.; the resulting temperature is found to be 100° F. Find the temperature of the furnace.

PRACTICAL EXERCISES.

1. Mix together equal weights (say, 100 grams or 50 grams each) of cold water and hot water, say, at 50° C., and find the resulting temperature; first pouring the cold

water into the hot, and then pouring back into the first vessel, so that both vessels (which should be of the same size) reach the same final temperature.

2. Mix unequal weights of hot and cold water, and tabulate in each case the heat given by the hot water (weight × fall in temperature), and the heat received by the cold water (weight × rise in temperature). Are the quantities nearly equal? As the numbers are large, the differences may be considerable.

3. Heat 50 or 100 grams of lead shot or small iron tacks in a test-tube surrounded with steam or boiling water until they reach 99° or 100° C.; transfer quickly to an equal weight of cold water, and find the rise of emperature (being careful that no hot water is transferred vith the metal). Compare the fall in temperature of the netal with the rise in temperature of the water, and draw onclusions as to their relative heat capacities.

(NOTE.—If the mixture is made in a copper calorimeter, he latter may be taken to absorb as much heat as $\frac{1}{10}$ its weight of water, and a correspondingly smaller quantity of cold water used.)

4. Convert a good Bunsen burner into a small furnace by surrounding the flame with two fireclay crucibles, with the bottoms knocked off; then place an iron nut on a wire triangle between the crucibles. When the nut is as hot as possible, quickly transfer it to a calorimeter containing about six or eight times its weight of cold water, and, by finding the rise of temperature, calculate the temperature of the iron.

(NOTE.—Be careful to stir up the water until a uniform temperature is reached, and do not read the temperature until it ceases to rise.)

5. Make a *rough* comparison of the specific heats of water and paraffin by taking the times (in minutes and seconds) required to heat equal weights of water and paraffin in a metal vessel heated by a constant flame,

until the temperature rises, say, 20° in each case, the liquids being stirred all the time. (Students should work in pairs, one acting as timekeeper.)

6. Make a more accurate determination of the specific heat of paraffin by placing 100 *grams* (not 100 c.c.) of it in a calorimeter, pouring in 100 grams of lead shot previously heated to 100° C., and noting the final temperature of the mixture. Taking the specific heat of lead as ·031, calculate the heat given out by the lead, and subtracting the heat received by the calorimeter, the remainder will be the heat received by the paraffin. Knowing the weight and rise of temperature of the paraffin, its specific heat can now be calculated.

CHAPTER XVIII

MELTING OR FUSION

It is well known that many solids, when heated to a sufficiently high temperature, melt, or change to the liquid state. The temperature at which this change takes place differs greatly with different substances. Thus, wax melts in hot water (*e.g.* beeswax melts at about 55° C.), whereas lead must be heated to 330° C., and cast-iron to about 1200° C.; and some metals, such as platinum, must be heated to a temperature higher than that of any ordinary furnace.

There are other substances, such as wood and coal, which do not melt at all, but, when heated, undergo other changes of a chemical nature.

Again, there are substances, such as charcoal and lime, so refractory that no known source of heat is sufficient to liquefy them.

On the other hand, there are many substances which are liquid at the ordinary temperature, but which become solid when sufficiently cooled. Thus, water forms solid ice below 0° C., and at this same temperature the solid melts and forms water again. The liquid metal mercury only solidifies at -39° C., and alcohol or spirit of wine must be cooled to a still lower temperature before it solidifies.

It may be shown by experiment that, for each substance, there is a definite temperature at which melting (in the case of solids), or solidification, or freezing (in the case of liquids), takes place. This temperature is called the **melting-point.** We have already seen that the melting-point of ice, or the freezing-point of water, has been chosen as one of the fixed temperatures for the construction of thermometer scales.

If we take some pieces of ice and pour hot water upon them, and, after stirring up the mixture for a few moments, we take the temperature, we shall find that it is still 0° C., in spite of the heat supplied by the hot water, but it will be observed that some of the ice has been melted in the process. This illustrates the important fact that, during the process of melting, the temperature remains constant, whatever be the temperature of the surroundings.

If we next mix some salt and broken ice, and test the temperature, we shall find it to be considerably below 0° C., in fact it may fall as low as -22° C., and at the same time it will be found that much of the ice is melted. This shows that impurities, such as salt, tend to lower the melting-point, and it explains why the salt water of the ocean is so much more difficult to freeze than fresh water.

There are some substances, such as butter, and sealing-wax, and glass, and wrought-iron, which do not pass suddenly from the solid to the liquid form at a definite temperature, but which, on heating, first become soft and plastic, then form very thick viscous liquids, and only at a considerably higher temperature do they become quite liquid. They pass, as it were, through a state intermediate between solid and liquid, and they cannot be said to have a definite melting-point.

This property is of great practical importance; thus, in the case of wrought-iron, when it is made red-hot it is sufficiently plastic to be shaped by the blows of a hammer, or by the rollers between which it is squeezed in a rolling-mill. Again, glass, which at the ordinary temperature is extremely brittle, when heated becomes soft enough to be bent, or moulded, or blown (by air pressure) into the required shape, at temperatures far below that at which it becomes quite fluid. A further advantage is that when in this semi-fluid condition, two pieces may be united into one by the process of welding; this is much used by the smith in working wrought-iron, and also by the glass-blower.

The change from the solid to the liquid state is always accompanied by a slight change of volume. Thus, ice occupies a somewhat greater space than the water from which it is formed, hence a pound of ice takes up rather more room than a pound of water, *i.e.* the density of the ice is smaller, and this is why ice floats on water.

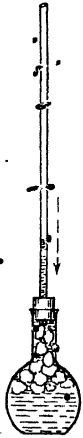

FIG. 83.

The change in volume is easily shown by putting some pieces of ice in a flask, filling up with water, and attaching a narrow tube. On placing the flask in warm water, it will be found that, as the ice melts, there is a continuous contraction indicated by the steady fall of the water in the stem.

The expansion which occurs when water freezes, although small in amount, takes place with very great force, and this accounts for the bursting of water-pipes, and the splitting of rocks, &c., in time of frost. It has been shown that a thick iron bomb-shell, when quite filled with water and securely plugged, can be burst by putting it out in a keen frost. It is quite easy to burst a bulb made of strong, thick glass, by filling with water and closing the end by melting the glass, and then placing it in a freezing mixture of ice and salt.

Many solids expand instead of contracting on melting. Thus, if we melt some paraffin wax, and drop a lump of the solid into the liquid portion, we shall find that it does not float like ice, but sinks to the bottom, showing that the liquid is less dense, and therefore occupies a greater volume than the solid from which it was formed.

The question of expansion or contraction on solidification is of considerable importance in making castings. Thus, if the liquid which fills the mould contracts on becoming solid, the solid will not completely fill all the corners and crevices of the mould, and the casting will be lacking in sharpness, and will not be an exact copy of the mould. On the other hand, if a slight expansion occurs on solidification, this will force the metal into every crevice, and a sharp casting will result.

Lead does not form sharp castings, because the molten metal contracts on solidification. This makes it unsuitable for casting the type used in printing, but, when alloyed with a little antimony, to form type metal, there is a slight expansion, and the necessary sharpness is obtained.

Latent Heat of Fusion.—We have already seen that when some hot water is mixed with ice the temperature quickly falls to 0° C. The heat of the hot water seems to disappear mysteriously, without raising the temperature of the ice in the slightest degree, but we must not forget that the hot water has produced the important effect of melting some of the ice, and the more hot water we use, or the higher the temperature of this water, the more ice will be melted.

It can be shown by experiment that, if we mix 1 gram of water at 80° C. and 1 gram of ice at 0° C. the whole of the ice is just melted, and we get 2 grams of water at 0° C.

On the other hand, if we mix 1 gram of water at 80° C. and 1 gram of *water* at 0° C., we shall obtain 2 grams of water at 40° C.

We may state the two results as follows:—

1 gram water at 80° + 1 gram *ice* at 0° C. = 2 grams water at 0° C.

1 gram water at 80° + 1 gram *water* at 0° C. = 2 grams water at 40° C.

In the first case the 1 gram of water is cooled to 0° C. (*i.e.* through 80° C.), hence it gives out 80 units of heat. These 80 units of heat must have passed to the ice, and have only caused the ice to melt without raising its temperature. The heat which is consumed in this way in melting a solid without raising its temperature is called the **latent heat of fusion.** Thus the latent heat of fusion of ice is 80. If we use the British thermal unit, the latent heat is expressed by the number 143. Thus 1 lb. of water in cooling 143° F. gives out just enough heat to melt 1 lb. of ice without raising its temperature.

The large quantity of heat used up in melting ice explains why it takes ice so long to melt after the thaw comes, because all this heat must be imparted to the ice by the air and surrounding objects before it can melt. It is well to note that exactly the same quantity of latent heat is set free when water solidifies, and this is why it takes so long for the ice on a pond to become thick, even when the frost is keen, because all this latent heat must be abstracted from the water in the process of freezing.

Solution.—It is well known that some solids, such as salt and sugar, when shaken up with water, are gradually liquefied and mix with the water. This process is quite different from that of melting, for the solid is in this case liquefied, not by heat, but by contact with the liquid water, although it is true that heat often assists the process.

This process is called **solution**; the solid is said to **dissolve** in the water, and the mixture is called a **solution**. The amount of solid which a given quantity of water can dissolve is limited, and when the limit is reached it forms a **saturated solution.** Hot water will generally dissolve more of the solid than cold water, and some solids dissolve more freely than others. A substance which will dissolve in water is said to be **soluble,** and one which will not dissolve is **insoluble.**

One important point of resemblance between dissolving and melting is that in both cases latent heat is consumed in the process. Every amateur photographer knows that when he dissolves "hypo" in water to make a strong solution the liquid becomes very cold; this is due to the latent heat used up in liquefying the solid. Some very soluble substances, such as ammonium nitrate, consume so much heat in dissolving, that the solution falls below 0° C., and such are sometimes used as "freezing mixtures."

SUMMARY.

1. Each substance has a definite melting-point.

2. Impurities lower the melting-point.

3. Some substances, as glass and wrought-iron, soften long before becoming liquid.

4. There is a *slight* change of volume in melting or freezing; thus ice has a greater volume than water.

The expansion occurs with great force.

Many substances contract on solidifying—this is bad in casting.

5. Latent heat is consumed when a solid melts.

To melt 1 gram of ice requires 80 heat units.

6. Solution is liquefaction by contact with a liquid solvent.

Substances may be classed as soluble and insoluble.

PRACTICAL EXERCISES.

1. Compare the temperature of pure melting ice with that of a mixture of pounded ice and salt.

2. Find the melting-point of paraffin wax by putting a little of it in a narrow and thin-walled glass tube, sealed at the bottom, and fastened beside the bulb of a thermometer. Fix the thermometer with the bulb in a beaker or flask of water whose temperature is gradually raised until the wax melts, as seen by its becoming clear.

3. Mix approximately equal weights of ice and boiling water, and find the resulting temperature. How many degrees is the hot water cooled? How many degrees is the melted ice heated? The difference is accounted for by the heat used in melting the ice. How many heat units do you conclude would be required to melt a gram of ice. (Note, that since the weights of ice and water are equal, the resulting temperature will be the same whether we use 1 gram of each or 100 of each, or any other weight.)

4. Find the fall in temperature when 10 grams of powdered washing soda are dissolved in 100 grams of water at about 50° C.

CHAPTER XIX

BOILING

THE change from the liquid state to that of gas or vapour, by the process of boiling, has several points of resemblance to that of melting. Thus each liquid has a definite **boiling-point**; *i.e.* the change takes place at a particular temperature. During the process the temperature remains steadily at this point, however strong the source of heat; the only effect of increasing the heat supply is to make the liquid change more quickly to vapour.

We may study the process by boiling some water in a glass flask. It will be seen that, as soon as the water begins to boil, the surface becomes disturbed by a kind of eruption, caused by the bubbles of steam, which are formed in the liquid, rising to the surface, and there bursting and escaping into the space above. The steam above the liquid is quite invisible, like air, but at the mouth of the flask there is a kind of cloud or mist, produced by the condensation of the steam into exceedingly small liquid drops, by contact with the cold air. A thermometer placed in the steam should indicate a temperature of 100° C.,[1] but if it dips in the liquid it may record a slightly higher temperature; thus, in determining boiling-points, the thermometer bulb should always be placed in the vapour, and not in the liquid.

If we boil salt water, we find that the boiling-point is higher than 100° C.; thus the presence of impurities *raises* the boiling-point.

The steam from the salt water very soon falls to 100° C., and it contains no salt, for if we **condense** it to water by cooling, the water so formed will be found to be free from salt. This process of boiling a liquid and condensing the vapour back to liquid in a separate vessel is called **distillation**, and is often used in purifying water and other liquids, the impurities being left behind in the boiler, or "retort," as it is called.

The condensation is generally effected by causing the vapour to pass through a long coiled tube (called a worm) lying in a tub, through which a stream of cold water flows.

The change in volume when solids melt is small, but when a liquid boils the expansion is enormous; thus, a cubic inch of water forms nearly a cubic foot (1728 cubic inches) of steam. This expansion, if resisted, takes place with enormous force. This force is usefully employed in driving steam-engines, but it may cause serious disasters in the way of boiler explosions unless the boilers are strong and provided with proper safety-valves.

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[1] Provided the atmospheric pressure as indicated by the barometer is normal (*i.e.* 30 inches, or, more accurately, 760 millimetres of mercury), and that the stem as well as the bulb of the thermometer is surrounded with steam.

Change of pressure has an important effect on the temperature at which a liquid boils; thus, if the pressure be increased, the boiling-point is raised, whereas, if the pressure be reduced, the liquid may be made to boil far below its normal boiling-point.

This point may be illustrated by a very simple and curious experiment. Water is first boiled in a strong flask until the steam drives out the air. The flask is then corked and inverted. If cold water be now poured on the outside of the flask, the water inside begins to boil, and the more cold water we pour upon it, the more boisterously does the boiling take place. The secret of this remarkable experiment is that there is no air in the flask, consequently, when we condense the steam in it, by the application of cold water, there is a partial vacuum created inside, and under the reduced pressure the water boils even in spite of its being cooled by the cold water.

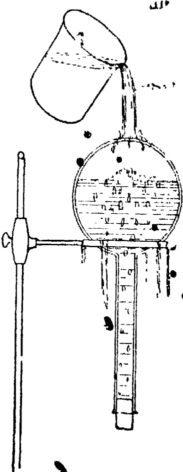

FIG. 84.

The reduction of pressure by an air-pump also causes water to boil below 100° C. It is well known to travellers that, on a high mountain, water boils far below the ordinary boiling-point, on account of the reduced pressure of the air.

On the other hand, in a steam-boiler, where steam is generated under considerable pressure, the boiling-point of the water is far above the ordinary boiling-point. Thus, under a pressure of 200 lbs. on the square inch, which is not uncommon in locomotive boilers, the boiling-point of water is 194° C. or 381° F.

Latent Heat of Steam.—When water is converted into steam a much greater quantity of heat disappears, or is consumed in the process, than in the melting of ice. Thus, whereas the latent heat of fusion of ice is 80, the latent heat of steam is no less than 537 (or 966 if we use British thermal units[1]). This means that it requires as much heat to convert 1 gram of steam at 100° C. into water at 100° C. as would be sufficient to heat 537 grams of water 1° C. This explains how it is that it does not take nearly so long to raise water from the ordinary temperature to the boiling-point as it does to cause all the water to boil away, for to heat 1 gram of water from the freezing to boiling-point only requires 100 units of heat (or 180 British units per pound of water), whereas to boil it away requires 537 units (or 966 British units per pound). Conversely, of course, a gram of steam at 100° C. would give out 537 units of heat in condensing to water at 100° C. The value of the latent heat of steam, which is of great importance to engineers, may be found by passing the steam into a weighed quantity of cold water, whose temperature is known. The steam condenses and imparts its latent heat to the water, and also a further quantity of heat, due to its cooling below 100° C. If we take the temperature of the water again, the rise indicates the quantity of heat given up by the steam, and by weighing it again, the gain in weight indicates the quantity of steam used.

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[1] The reason why the latent heat in British units is greater is that in the British unit we use the degree Fahrenheit, which is shorter than the Centigrade degree in the ratio of 5 : 9. Thus 537 is $\frac{5}{9}$ of 966.

Examples :

Steam is passed into 100 grams of water at 15° C. until the temperature reaches 35° C. The gain in weight due to the steam is 3·32 grams. Find the latent heat of steam.

The 100 grams of water are heated 20° C., and therefore receive

100 × 20 = 2000 units of heat.

This heat comes from the 3·32 grams of steam.

Hence the heat from each gram of steam = 2000 ÷ 3·32 = 602·4 units.

But the condensed steam is cooled from 100° to 35°; *i.e.* 65° fall, by which each gram would give out 65 heat units.

Hence the *latent* heat = 602·4 − 65 = 537·4.

The high latent heat of steam explains the large quantity of condensing water required in a condensing engine.

Suppose that condensing water is supplied at 60° F. and leaves the hot well at 100° F.; find how many gallons of water will be required for each pound of steam used.

Using British units, we find that each pound of steam gives out 966 heat units of latent heat in condensing,[1] and also 112 units in cooling from 212° F. to 100° F.

Hence the total heat given out per pound of steam = 1078 units.

This heat goes to the water, and heats it from 60° F. to 100° F., *i.e.* 40° rise.

Hence each pound of water receives 40 units of heat.

Therefore to take up the 1078 heat units given by the steam requires 1078 ÷ 40 = 26·9 lbs = 2·69 gallons (since a gallon of water = 10 lbs).

EVAPORATION.

It is well known that a pool of water gradually dries up, even at the ordinary temperature. This is due to the water being converted into vapour by a silent and invisible process, called evaporation. The change is, in reality, the same as that which occurs when water boils, only it takes place under very different conditions. Whereas boiling only takes place at a particular temperature, called the boiling-point, evaporation goes on at all ordinary temperatures. Again, in the case of boiling, there is a visible eruption of vapour at the surface, whereas in evaporation the surface is undisturbed, simply because the vapour is only produced at the surface, and not in the interior of the liquid, as in the case of boiling. This being so, it is easy to understand why the rate of evaporation depends on the extent of surface; thus water evaporates more quickly when exposed in a shallow plate than when the same quantity is placed in a cup or other narrow vessel.

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[1] Assuming that condensation occurs at atmospheric pressure.

The rate of evaporation depends on several other things; thus it is well known that wet clothes dry more rapidly on some days than others. Other things being equal, evaporation goes on more rapidly when the temperature is high than when it is low. But even when the temperature is the same, evaporation may take place more quickly on one day than another. This depends on the fact that, at a particular temperature, the air will only retain a limited quantity of vapour; when this limit is reached, the air is said to be saturated, and evaporation then ceases entirely.

The rate of evaporation depends on the dryness or dampness of the air; *i.e.* how far it is removed from the point of saturation.

Wind promotes evaporation by constantly removing the air near the source of moisture before it has time to become saturated, and replacing it by fresh air further removed from the saturation point.

It requires more moisture to saturate the air when warm than when the temperature is low, and hence a rise of temperature promotes evaporation.

If warm air, containing a considerable quantity of vapour, be cooled, a temperature will at last be reached at which this vapour will be sufficient to saturate it, and any further cooling will result in some of the moisture condensing to the liquid form. The temperature at which this condensation begins is called the **dew-point.**

Since condensation will take place more readily when the air contains much vapour, it follows that the dew-point will be higher when the air is damp than when it is dry.

This condensation of moisture in the air takes place in various forms: thus when it condenses in the upper regions, clouds are formed consisting of masses of minute water-drops; when these drops become larger they descend as rain. Condensation near the ground level produces mist or fog, and condensation on the solid objects on the ground gives rise to dew, or if the temperature be below the freezing-point, hoar-frost is formed.

Other liquids besides water evaporate on exposure; some of them, *e.g.* spirit of wine or alcohol and turpentine, dry up more quickly than water; others evaporate more slowly, *e.g.* paraffin oil; and others, again, not at all, *e.g.* many oils, such as olive oil. Generally speaking, those liquids which have a low boiling-point evaporate more rapidly, or are more *volatile*, as we say, than those with a high boiling-point. Those liquids which do not evaporate at all appreciably are said to be non-volatile.

Latent Heat of Evaporation.—The process of slow evaporation at the ordinary temperature is accompanied by an absorption of latent heat, no less than the process of boiling. The well-known chilly feeling which results from standing in a draught with damp clothing is caused by the absorption of latent heat of evaporation. When the ground is watered in hot weather, it not only lays the dust, but spreads a pleasant coolness, due to the evaporation of the water.

The rapid evaporation of a very volatile liquid, such as ether, absorbs so much heat that ice is readily produced by placing a thin vessel containing water in contact with it.

Hygrometers.—Since the amount of water vapour in the air varies greatly, it is desirable to have some means of finding how much is present at any given time. An instrument for this purpose is called a hygrometer. Most of these depend on finding the dew-point, *i.e.* the temperature at which condensation begins. We have already seen that when there is much vapour present, the dew-point is higher than when there is little.

The hygrometer most commonly used depends on the two facts: (1) that the rate of evaporation is greater the farther the air is removed from the point of saturation, *i.e.* the drier it is, and (2) that when evaporation takes place, latent heat is consumed. The instrument consists simply of two ordinary thermometers, the bulb of one of them being covered with muslin, which is kept constantly wet by a kind of lamp-wick dipping in a vessel of water. As evaporation takes place from this wet bulb, heat is consumed, and hence it shows a lower temperature than the dry bulb. The drier the air, the more rapid will be the evaporation and the consequent consumption of heat, and hence the *difference* in the readings of the wet and dry bulbs will be greater when the air is dry than when it is moist, and if at any time the air is quite saturated, evaporation will cease, and both thermometers will show the same reading. The actual amount of moisture corresponding to any two readings of the wet and dry bulbs is found by consulting a set of tables. Such wet and dry bulb hygrometers are sometimes used in cotton-weaving sheds, in which the dampness of the air has an important effect on the weaving process. When the air becomes too dry, it is often artificially humidified by means of steam or water spray.

SUMMARY.

1. Each liquid has a definite boiling-point.

2. One cubic inch of water forms about 1 cubic foot of steam.

3. Decrease of pressure lowers the boiling-point, increase of pressure raises it.

4. Latent heat of steam = 537 (or 966 in British units). Hence the large amount of cold water required to condense steam.

5. Evaporation takes place silently at the ordinary temperature.

6. Rate of evaporation depends on (*a*) extent of surface, (*b*) temperature, (*c*) dryness of the air, (*d*) presence or absence of wind.

7. The dew-point is the temperature at which the moisture in the air begins to condense.

8. A hygrometer measures the dampness of the air. A simple form consists of wet and dry bulb thermometers.

EXERCISES.

1. How much steam at 100° C. must be blown into a gallon of water at 10° C. to just raise it up to 100° C.?

2. How many units of heat would be necessary to convert 50 grams of ice at 0° C. into steam at 100° C.?

3. A steam-engine uses 5 lbs. of steam per minute. How much condensing water must be supplied at 50° F. so that it may not be raised above 100° F.?

4. If it takes 5 minutes to raise a pint of water from 10° C. to the boiling-point, how long will it take to boil dry if the heat is received at a constant rate

PRACTICAL EXERCISES.

1. Test the 100° C. mark on a thermometer by surrounding it with steam from boiling water contained in

a flask with a neck long enough to cover the thermometer stem.

Try the boiling-point of a solution of salt (*a*) with the thermometer in the liquid, (*b*) with the thermometer in the steam.

2. Find the boiling-points of one or more liquids, such as methylated spirit, by using a small distilling flask.

3. Distil a sample of pure water from a solution of some coloured salt (say, copper sulphate)

4. To get a *rough* idea of the heat consumed in producing steam, heat some cold water in a metal vessel with a small steady flame, and find the time required to raise it up to boiling-point, and the time required to boil it completely away. Assuming that heat is received at the same rate all the time, compare the quantity of heat required to heat the water, say, from 15° to 100° C. (*i.e.* 85°), with the heat required to convert it to steam.

(*N.B.*—The water loses heat more quickly when hot than when cold, hence the comparison can only be a rough and ready one.)

5. Find the latent heat of steam by passing steam *rapidly* from a glass or metal boiler into a flask containing a weighed quantity of cold water (from 100 to 250 c.c.), whose temperature is known. Let the temperature rise some 20° or 30°, then withdraw the steam tube. After shaking the flask round to equalise the temperature, read the thermometer and then carefully weigh the flask to find the weight of steam condensed.

The method of calculation is indicated in the chapter. A simple form of trap should be used on the steam pipe, just above the cold water flask, to arrest any water due to spray or condensed steam.

6. Cover a thermometer bulb with a piece of damp rag, and compare its reading with that of a dry bulb thermometer.

CHAPTER XX

CONDUCTION, CONVECTION, AND RADIATION

THERE are three distinct processes by which heat can pass from one place to another. These three modes of transfer may be compared roughly to three ways in which water could be transferred from a pond to a burning building. First imagine a chain of men passing buckets of water from hand to hand; this corresponds to conduction, in which heat is passed from particle to particle of some conducting substance, as when heat passes along a poker, one end of which is stuck in the fire. Secondly, imagine that the men, instead of passing the buckets of water from hand to hand, actually carry the buckets from the pond to the building and return with the empty buckets, so that we have a continuous stream of men with full buckets in one direction and a return stream of men with empty buckets; this corresponds roughly to the convection of heat by moving currents of a liquid or gas; but, as we shall see later, the currents are maintained by the effect of the heat itself in causing expansion of the liquid or gas. Lastly, we may imagine the water to be projected or thrown over the intervening space in a continuous jet, so that the transfer is effected independently of any intervening men; this corresponds very roughly to the radiation of heat, in which it is, as it were, thrown out by a hot body, only, unlike the jet of water, the heat is thrown out in all directions. An example of this is afforded when heat reaches the earth from the sun, or when we fee the heat thrown out by a fire when we stand some distance away.

Conduction.—Common experience teaches us that some substances conduct heat more readily than others; thus, if we place a poker and a wooden stick side by side in the fire we shall find that the handle end of the poker soon gets uncomfortably warm, whereas, while one end of the wood burns away the other end scarcely becomes appreciably warm. We say that the iron is a fairly good conductor of heat, whereas the wood is a bad conductor, or, as it is sometimes called, a non-conductor.

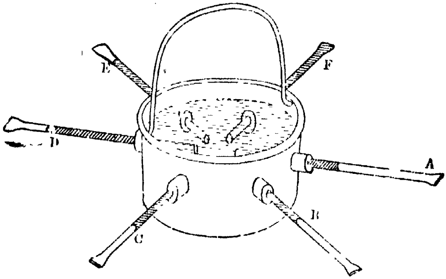

FIG. 85.

In order to compare the conducting power of different metals, we may use the apparatus shown in the figure, in which rods of the different metals project through the sides of a pan containing hot water, or heated oil. The projecting portions are coated with wax, and as the heat travels along the bars the wax melts. The wax will melt farthest along that bar which is the best conductor (F in the figure), and the melting will extend for the shortest distance along the worst conductor (A in the figure).

Silver is the best conductor, and, if we represent its conductivity by 100, that of copper will be about 75, and that of iron from 10 to 15. Thus it appears that copper is a much better conductor than iron; hence it is very suitable for use in the firebox and tubes of a locomotive boiler, to facilitate the transfer of heat from the fire and hot furnace gases to the water.

In some cases it is important to stop the removal of heat as far as possible by the use of non-conductors. Most porous and fibrous substances, such as fur, wool, feathers, straw, wood-shavings, and fibrous asbestos, are bad conductors on account of the air entangled in the pores and between the fibres, air and other gases being exceedingly bad conductors.

The warmth of clothing made of fur or woollen materials is accounted for in this way, the natural heat of the body being kept in by these non-conducting substances.

The wrapping of pumps and stand pipes with straw, and the packing round water cisterns with sawdust or shavings, as a protective against frost, is another useful application of non-conductors.

Steam boilers and steam pipes and cylinders are frequently surrounded with some special porous composition or asbestos packing to reduce the loss of heat.

Convection.—It has already been pointed out that when a liquid is heated, the expansion which takes place causes a decrease in density, since there is a smaller quantity of liquid in a given space after the expansion. This is sometimes expressed by saying that the liquid becomes lighter when heated, but such a statement is rather misleading. Thus, if we take a pound of water and heat it, it will still weigh a pound; but if we take, say, a gallon of cold water, which weighs 10 lbs., and heat it, it will expand so as to form more than a gallon, and if we throw away the excess and weigh just a gallon of the hot water, it will of course weigh less than 10 lbs., because there is now less water in the gallon measure than before.

The correct way of stating the case is that the hot water is less dense or *specifically* lighter, than the cold water.

The consequence of this is that if water contained in a kettle or in a steam boiler be heated from below, the heated water tends to rise and the colder water descends to take its place, so that a continuous circulation is set up and the heat is conveyed and distributed by the currents of warm water. This is the process called convection. The ascending and descending streams can be made visible if the water be heated in a glass vessel and some heavy sawdust or some fragments of colouring matter be introduced to trace out the currents. It will be observed, that the currents are only set up when the water is heated from *below*; if the heat be supplied at the top the heated water will remain at the top. As water is a very bad conductor, these convection currents are of great importance in distributing heat in a mass of water, and in some steam boilers there is an arrangement of tubes to promote the circulation.

These convection currents are also very usefully applied in the heating of buildings by hot-water pipes. The heat is supplied by a furnace and boiler at a low level: the heated water rises by a pipe which leaves the top of the boiler, and after circulating through the pipes and radiators flows at a lower temperature by a return pipe which enters the lower part of the boiler. In a similar manner the heat supplied to the boiler at the back of a kitchen fire-grate is conveyed to the storage cylinder from which the supply of hot water for domestic purposes is drawn.

Since air is greatly expanded by heat, it follows that convection currents are readily set up whenever air is heated more in one place than another. The draught in a chimney is caused by the upward tendency of the heated column of air. The longer the chimney and the hotter the gases it contains the more powerful will be the draught.

The formation of a convection current is easily illustrated by the arrangement shown in the figure. A cigar-box with two holes in the bottom is inverted and a glass chimney placed over each hole. A lighted candle burns under one of the holes, and by means of some smoke from smouldering paper it can be shown that there is an upward current in the chimney over the candle and a downward current in the other.

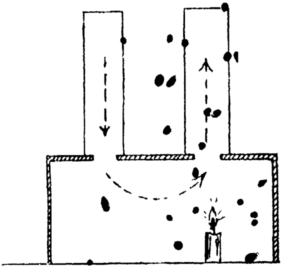

FIG. 86

A similar arrangement on a large scale is sometimes used in the ventilation of coal-mines, a fire being placed at the bottom of one of the shafts, causing an updraught which carries away the foul air of the mine, a constant stream of fresh air descending by a second shaft.

Convection currents are often used in the ventilation of public buildings, outlets being provided at the highest part of the roof by which the air which has been rendered foul and also heated by the presence of people or burning gas, escapes from the building, its place being taken by fresh air admitted from below.

Winds are, for the most part, to be considered as natural convection currents on a large scale caused by the unequal heating of the atmosphere at different parts.

Radiation.—Although the sun is about 92 millions of miles distant, and there is apparently no material substance in the intervening space, the earth receives a constant supply of light and heat from it.

Light and heat also stream out in all directions from a fire or from a red-hot mass of metal. Even if the mass of metal be not hot enough to be visible in the dark, it may still send out heat. This streaming out of light or heat, or both, from a hot body is called radiation.

There are many points of resemblance between light and radiant heat;[1] thus when we stand in the shade we escape, to some extent at least, from both the intense light and the heat of the sun. Again, both light and heat can be reflected or thrown back by a suitable surface, and whereas some substances, such as air and water, allow radiation to pass through them freely, there are others, such as wood, metal, and stone, which completely block their passage.

It should be noted, that when radiant heat passes through a medium such as dry air, it does not heat it; it is only when the radiations are absorbed, that the heating effect is perceptible. Thus an object placed in front of a fire may get much hotter than the air between it and the fire.

Substances differ greatly in their power of absorbing radiant heat. Thus it is well known that light-coloured clothing absorbs the sun's rays much less than dark clothing, and a bright shining metal surface absorbs heat less than a dull black surface.

There are corresponding differences in the power of radiation, or throwing off heat; thus a bright metal teapot loses heat by radiation much less quickly than one with a black, smoky, metal surface. In fact, good absorbers of radiant heat are themselves good radiators, and bad absorbers are bad radiators.

Since radiation takes place only from the surface of a body, it is clear that the rate of radiation will depend on the extent of surface, hence hot-water radiators are purposely made to expose a large surface, and in some cases projections called gills are cast on to the hot-water pipes simply to provide additional radiating surface.

---

[1] The fact is, light and radiant heat are simply different manifestations of the same phenomenon, but whereas light is only perceived by the eye, radiant heat affects all parts of the body.

Glass behaves in a peculiar way towards radiations; hus, while we can feel both the light and heat of the sun hrough a window, a sheet of glass forms a very effective ire-screen. The explanation is that, whereas glass is ransparent to ordinary light rays, it does not permit dark heat rays to pass through. Thus, a greenhouse acts as a kind of heat trap, allowing the heat and light of the sun to enter freely, but stopping the loss of heat by radiation from the objects within, since these radiations consist entirely of dark heat rays.

SUMMARY.

1. In conduction the heat is passed from particle to particle.

2. In convection the heat is carried by moving particles of liquid or gas.

3. In radiation the heat is, as it were, thrown out in all directions.

4. Metals are good conductors (copper and silver the best).

5. Porous and fibrous bodies are bad conductors, and often used to keep heat in.

6. Convection occurs when a liquid is heated from *below*, because the heated liquid is less dense, and therefore rises.

7. Hot-water systems distribute heat by convection.

8. Convection currents in air are useful for ventilation.

9. Radiant heat greatly resembles light.

10. The rate of radiation depends on the extent and nature of the surface. Good radiators are also good absorbers.

11. Glass absorbs dark heat rays.

PRACTICAL EXERCISES.

1. Examine the convection currents in water by taking a fairly wide beaker of water and heating it at one side,

the currents being traced by putting in a crystal of permanganate, or of a suitable dye, or some heavy sawdust. Notice both the ascending and descending streams.

2. Examine convection currents in air by means of cigar-box and a couple of lamp chimneys, as described in the foregoing chapter.

3. Twist together pieces of iron and copper wire of equal thickness, cover the ends with paraffin wax, and heat the junction with a gas flame, noticing how far along each wire the wax melts. Which do you conclude is the better conductor?

4. Hold a piece of glass rod or tubing in the flame until the end melts. Does the other end become very hot? Do you consider glass a good conductor?

5. Take two equal flasks full of hot water (at the same temperature); pack one in a chalk box filled with saw-dust, and leave the other exposed. Examine the temperature of each at intervals of 15 minutes. Does the water cool at the same rate when it is colder as it did when hot? Does the sawdust retard the cooling?

6. Take two equal copper vessels, polish one and smoke the other, fill up with hot water (both at the same temperature), and cover with a wooden or cork lid carrying a thermometer, and let each stand on a flat cork, or three small corks. Notice the rate of cooling in each case, and see whether the black or bright surface radiates heat more quickly.

ANSWERS

CHAPTER II

1. 25·4
2. ·189 in.
3. 1000 kilograms
4. 50 miles per hour (approx.)
5. 52½ sq. ft.
6. 10 miles per hour
7. 1520 sq. in.

CHAPTER III

1. 3 cub. ft. or 5184 cub. in.
2. 1320 sq. in.
3. 3111·[illegible] cub. in.
4. 1810·5 cub. in.
5. 121½ cub. in.; 38400 bricks
6. 17325 gallons

CHAPTER IV

1. 525 lbs.
2. 525 lbs.
3. 145·8 lbs. and 149·7 lbs.
4. ·12 in.
5. 36·76 lbs.

CHAPTER V

3. 35 lbs.
4. 28 in., 36 lbs., ¼ in.
5. 32 in., 30 lbs.

CHAPTER VI

1. 1,120,000 foot-pounds
2. 20 foot-tons
3. 2678·5 foot-tons; 3·03
4. 49,[illegible]07 foot-tons
5. 466·6 foot-pounds
6. 3000 foot-tons
7. 2640 foot-tons

CHAPTER VII

1. 2½ revolutions; 70 gea
2. 27; 58 10/11 turns
3. 14; 178 2/11 turns
4. 160 revolutions per mi
5. 15 in.

CHAPTER VIII

1. 4; 70 per cent
2. 28; 20 lbs.
3. 7½ lbs.

CHAPTER I

1. 1½ tons
2. 66 ft.
3. ·074 in.
4. 5·39 tons
5. 502·9

CHAPTER X

1. 1·583 cwt. and ·916 cwt.
2. 1·916 cwt. and 1·583 cwt.
3. 3½ feet from A

CHAPTER XI

1. 20 lbs.
2. 5 lbs.; 8·66 lbs.
3. 1 cwt.; 1·73 cwt.

CHAPTER XII

1. 72,000 kilograms
2. 8·68 lbs. per sq. in.
3. 217 lbs. per sq. in.
4. 279 tons ; 6·97 tons
5. ·82

CHAPTER XIII

1. 2310 lbs.
2. 16 lbs. per sq. in.
3. 3·2 cub. ft.
4. 3278·7 cub. ft.
5. 1.075 grams

CHAPTER XV

1. 2372° F.
2. 2732° F.; 617° F.; 455° F.
3. −0·4° F.
4. 6·6° F.
5. 15·5° C.
6. 176·6° C

CHAPTER XVI

1. 13·94 in.
2. ·366 in.
3. 35·99 in.; 36·055 in.
4. 73·2 c.c.
5. 2·6 cub. ft.

CHAPTER XVII

1. 1400 units
2. 85,000 calories
3. 4·48° C.
4. 9·5 grams
5. 46·4 gals.
6. 2·64 gals.
7. 2332° F.

CHAPTER XIX

1. 1·67 lbs.
2. 35,850 units
3. 10·78 gals. per min.
4. 29·8 min.

INDEX

ABSOLUTE zero of temperature, 138
Absorption of radiant heat, 170
Archimedes, principle of, 31, 106
Area, 11
—— of triangle, 12
—— of circle, 14
—— of ring, 15
Atmosphere, pressure of, 111

BAROMETER, 113
Bell syphon, 12
Belt pulleys, 56
Block tackle, 66
Boiling, 155
Boyle's law, 115
British thermal unit, 143
British units and metric equivalents, 11
Burette,
Bursting by ice, 151

CALLIPERS, 13
Calorie, 14
Calorimetry, 142
Capstan, 56
Centigrade thermometer, 126
Centigram, 10
Centimetre, 9
Centre of gravity, 87
Change-wheels, 62
Circle, 12
Circumference, 13
Clock (train of wheels), 62
Coefficient of expansion, 130
Cog-wheels, 58
Cohesion,
Composition of forces, 9
Compressibility of gases, 7
Condenser (water required), 159
Conduction of heat, 166
Cone (volume), 22
Conservation of energy, 51
Convection, 167
Crab, 60
Crane, 9
Crow-bar, 38
Cylinder (volume), 21

DENSITY, 28
Dew-point, 160
Diameter of circle,
Differential air thermometer
—— pulley, 66
Distillation, 156
Door, leverage of, 43
Ductility

EFFICIENCY of machin
Elasticity (solids), 4
—— of gases, 7, 14
Energy, 50
Equilibrium of forces,
Evaporation, 159
Expansion (solids), 129
—— (liquids), 134
—— (gases), 137
—— (coefficient), 130
—— (force of), 133

FAHRENHEIT thermometer,
Fluidity, 5
Fluid pressure, 100
Foot-pound, 49
—— ton, 49
Force, 1, 47
—— pump, 121
Fulcrum, 37
Furnace, temperature of, 14
Fusion, 140

GALLON, 10
Gases (properties), 6
Gradient of incline, 72
Gram, 10
Gravity, 2

HEAD of water, 10
Hot water apparatus, 168
Hydraulic press, 10
Hydrometer, 107
Hygrometer, 162

IDLE wheels, 59
Inclined plane, 73, 9

KILOGRAM, 10
Kilometre, 9
Kinetic energy, 50

LATENT heat of fusion, 153
—— heat of solution, 154
—— heat of steam, 158
Lathe, 57, 62, 78
Levers, 37
—— three orders, 45
Liquids, properties of, 4, 5
Litre, 10
Loaded beam, 82, 84

MAGDEBURG hemispheres, 112
Malleability, 5
Mass, 47
Matter, 1
Measuring cylinder, 23
Mechanical advantage, 38
Melting point, 150
Metric system, 8
—— equivalents, 11
Micrometer screw gauge, 78
Milligram, 10
Millimetre,
Moment of force, 42

NON-CONDUCTORS, 160

OAR as a lever, 43

PARALLEL forces, 85
Parallelogram of forces, 94
Perpetual motion, 51
Pile-driver, 50
Pint, 23
Pipette, 119
Pitch of screw, 77
—— of teeth of wheel, 58
Potential energy, 50
Pressure of liquids, 100
—— of atmosphere, 111
Principle of Archimedes, 34, 106
—— of equal work, 40
Prisms, volume of, 20
Properties of matter, 2
Pulley (blocks), 65
Pulleys (belt), 56
Pumps, 120

RADIATION, 169
Reaction at supports of beam, 82
Rectangle, area of, 11
Resolution of forces, 95
Resultant of parallel forces, 86
—— of oblique forces, 94
Rigidity, 4
Roof-truss, 98

SAFETY-VALVE (lever), 44
Screw, 76
—— gauge, 78
—— jack, 79
Solids, 3
Solution, 153
Specific gravity, 29-31
—— heat, 144
Sphere (volume), 22
Stability, 88
Steelyard, 41
Syphon, 121

TEMPERATURE, 124
Thermal unit, 143
Thermometer, 125
—— scales, 126
Toothed wheels, 58
Train of wheels, 59
Triangle (area), 12
—— of forces, 90

U-TUBE (for densities), 108
—— (for Boyle's law), 114

VISCOSITY, 6
Volume, 18
—— of prisms, 21
—— of cylinder, 21
—— of cone and sphere, 22
—— of irregular solids, 25
Volumometer, 25

WATER—weight of 1 c.c., 10
—— weight of gallon, 22
—— weight of cubic foot, 27
Wedge, 75
Weight (units of), 10
—— distinguished from mass,
Wheel and axle, 53
Winch, 60
Windlass, 55
Work, unit of, 49
—— principle of equal, 40

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